PI – UNIT I –CHAPTER 3

DYNAMICS OF SYSTEM OF PARTICLES

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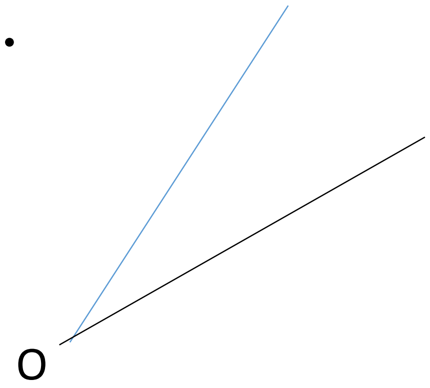
Dynamics of a particle

- Dynamics – motion of the particle – quantities defined for dynamics – displacement, velocity, acceleration, Force acting on the particle etc.
- Force acting on the particle $\vec{F} = \frac{d\vec{p}}{dt}$. where $\vec{p}$ is the momentum of the particle.
- If there is no force acting on the particle $\vec{F} = 0 = \frac{d\vec{p}}{dt}$, then momentum $\vec{p}$ of the particle is constant in time. This means, initial momentum = final momentum so Linear momentum is conserved.
- **Principle of conservation of momentum states that If there is no force acting on the particle , its linear momentum is conserved.**
- Rotation of the particle – Angular momentum – moment of momentum
- Torque – Moment of the force.
- Conservation of angular momentum is stated as if there is no torque acting on the particle then its angular momentum is conserved.
- Torque =$(\vec{\tau}) = \overrightarrow{N} = \vec{r} \times \vec{F}$ and Angular momentum $\vec{L} = \vec{r} \times \vec{p}$. And $\vec{N} = \frac{d\vec{L}}{dt}$
- Total energy is also conserved. E =T+V (T- K.E. , V- P.E.

INTRODUCTION to System of particles

1. System of particles- rigid body – several masses m_1, m_2 ,m_3 ……m_n with position vectors $\mathbf{r_1}$, $\mathbf{r_2}$, $\mathbf{r_3}$ …….$\mathbf{r_n}$ as shown in fig.1.
2. In the fig. we observe that o



Expression for conservation of linear momentum

- Now we write

Concept of center of mass

1. A body is said to be rigid body if it is non deformable, i.e. when external force is applied then it does not produce any displacement of the particles relative to each other.

2. In translational motion almost all particles of the system has same displacement, but in rotational or vibrational motion every particle may have different path. Ex. If a based ball is tossed up then every particle may follow same path but if baseball-bat is tossed then every particle may follow different path.

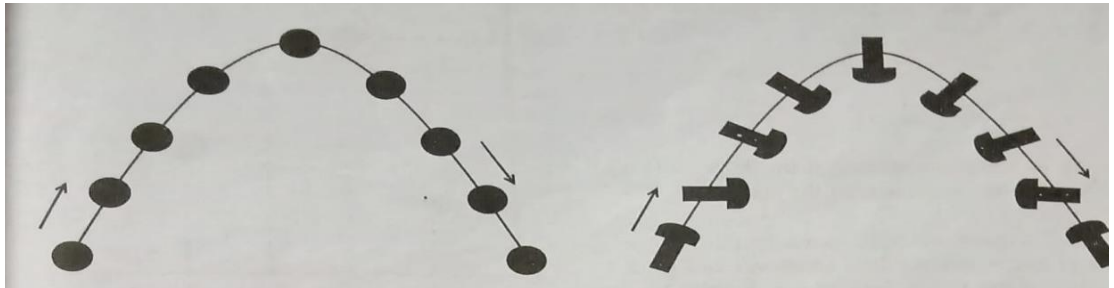


Fig. 2.1(a): Motion of ball

Fig. 2.1(b): Motion of baseball bat

(15)

3. Bat is not a particle but system of particles. each particle follow its own path. But it is observed that there is one point that follows a simple parabolic path where total mass of the body is supposed to be concentrated.

4. **This point is called as center of mass of the system.**

The center of mass of the body /system is defined as a point inside the body where the whole mass of the body is supposed to be concentrated.

CENTER Of MASS

- To define the center of mass, consider system of the particles for which masses of the particles are m_1 , m_2 ,m_3 ….. m_n have position vectors, $\mathbf{r_1}$, $\mathbf{r_2}$, $\mathbf{r_3}$ …….$\mathbf{r_n}$

Motion of center of mass

- Consider center of mass for system of the particle

Angular momentum of system of particles

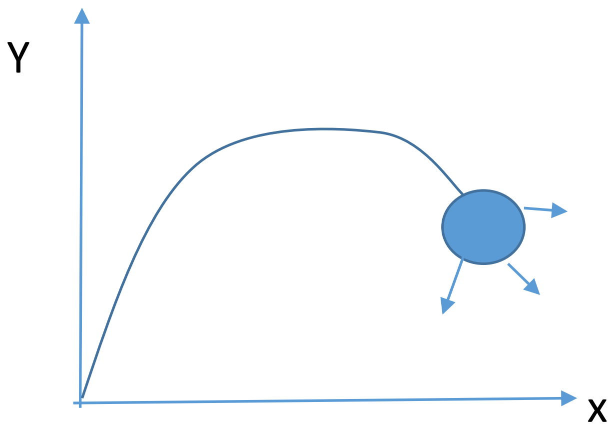
- Angular momentum of a particle - $\vec{L} = \vec{r} \times \vec{p}$ where p – linear momentum = mv (m- mass of a particle, v – velocity of a particle). $\vec{r}$ is distance of the particle from axis of rotation or displacement vector.
- Angular momentum of system of particles
- $\vec{L} = \overrightarrow{L_1} + \overrightarrow{L_2} + \overrightarrow{L_3} + \ldots\ldots\ldots\ldots + \overrightarrow{L_n} = \sum_{i=1}^{n} L_i$
- To find the exact expression for total angular momentum of system of the particles consider the particles of the system having masses m_1 , m_2 ,m_3 ….. m_n have position vectors, $\mathbf{r_1}$, $\mathbf{r_2}$, $\mathbf{r_3}$ …….$\mathbf{r_n}$
- Let the particles have momenta as p_1 , p_2 ,p_3 … ..p_n etc. For i^{th} particle let us consider angular momentum is given by $\overrightarrow{L_i} = \overrightarrow{r_i} \times \overrightarrow{p_i}$
- $\sum_{i=1}^{n} L_i = \sum_{i=1}^{n} \overrightarrow{r_i} \times \overrightarrow{p_i}$

Relation between angular momentum and torque for the system of particles.

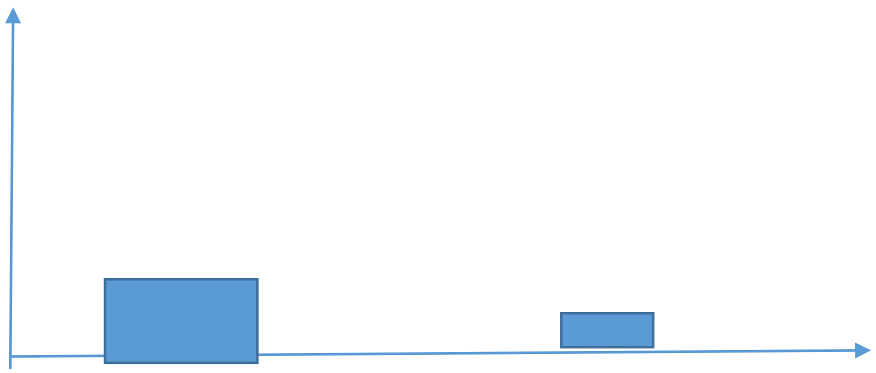
- Consider a system of particles having masses of the particles are m_1 , m_2 ,m_3 ….. m_n have position vectors, $\mathbf{r_1}$, $\mathbf{r_2}$, $\mathbf{r_3}$ …….$\mathbf{r_n}$. Let us consider $\mathbf{r_i}$ is the position vector and $\mathbf{p_i}$ is the linear momentum of the particle then angular momentum of the particle is written as $\overrightarrow{L_i} = \overrightarrow{r_i} \times \overrightarrow{p_i}$
- Total angular momentum for system is given by $\overrightarrow{L} = \sum_{i=1}^{n} \overrightarrow{L_i} = \sum_{i=1}^{n} \overrightarrow{r_i} \times \overrightarrow{p_i}$.
- Now consider $\vec{\tau} = \frac{d\vec{L}}{dt} = \frac{d}{dt} \sum_{i=1}^{n} \overrightarrow{r_i} \times \overrightarrow{p_i} = \sum_{i=1}^{n} \frac{d\overrightarrow{r_i}}{dt} \times \overrightarrow{p_i} + \sum_{i=1}^{n} \overrightarrow{r_i} \times \frac{\overrightarrow{dp_i}}{dt}$
- $\therefore \vec{\tau} = \sum_{i=1}^{n} \overrightarrow{\dot{r}_i} \times \overrightarrow{p_i} + \sum_{i=1}^{n} \overrightarrow{r_i} \times \overrightarrow{\dot{p}_i}$ .Consider the first term i.e.
- $\sum_{i=1}^{n} \overrightarrow{\dot{r}_i} \times \overrightarrow{p_i} = \sum_{i=1}^{n} \overrightarrow{\dot{r}_i} \times m_i \overrightarrow{v_i} = \sum_{i=1}^{n} \overrightarrow{v_i} \times m_i \overrightarrow{v_i} = 0$.
- Thus $\vec{\tau} = \sum_{i=1}^{n} \overrightarrow{r_i} \times \overrightarrow{\dot{p}_i}$. Let $\overrightarrow{\dot{p}_i}$ is the total force acting on the i[th] particle by definition
- $\vec{F} = \overrightarrow{F^e} + \sum_{j}^{n} \overrightarrow{F_{ij}^I}$. Substituting this in above equation, we get
- $\vec{\tau} = \sum_{i=1}^{n} \overrightarrow{r_i} \times \vec{F} = \sum_{i=1}^{n} \overrightarrow{r_i} \times \left[\overrightarrow{F^e} + \sum_{j}^{n} \overrightarrow{F_{ij}^I} \right] = \sum_{i=1}^{n} \overrightarrow{r_i} \times \overrightarrow{F^e} + \sum_{i=1}^{n} \overrightarrow{r_i} \times \left[\sum_{j}^{n} \overrightarrow{F_{ij}^I} \right]$
- Second term is sum of internal force acting on the particle which is equal to zero.

Some applications of conservation of momentum

- 1. Bomb explosion (projectile motion): A Projectile which is in a shell that explodes while in a flight as shown in figure



2. System of two blocks of masses m_1 , m_2 are tied or coupled by a spring resting on a horizontal frictionless table.
When blocks are pulled and released then initially, there is no momentum but after the release of force, they move in opposite direction as shown in fig.
Total initial momentum is zero but after their release their total final momentum should also be zero

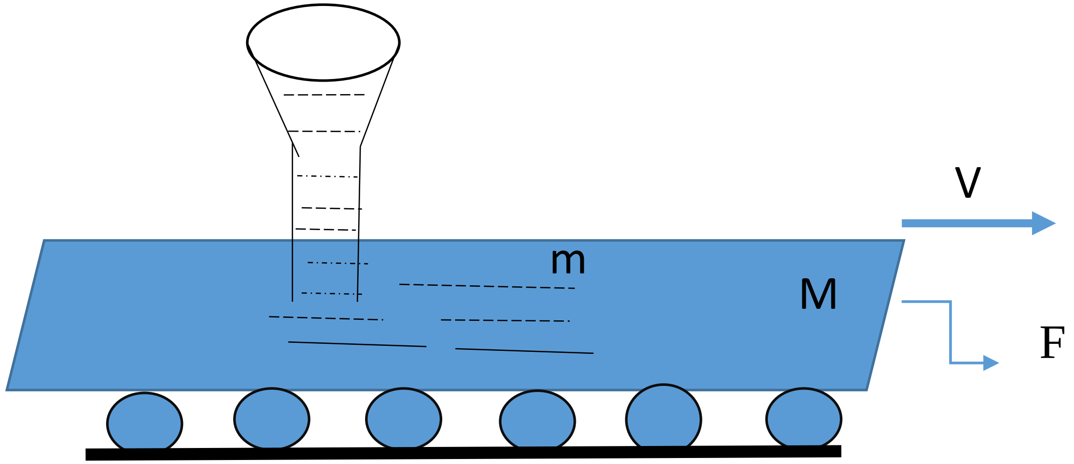


Motion of system with variable mass

- Examples: Rain drops falling through the cloud will gain mass as it reaches to the ground .
- Motion of Rocket: Rocket is launched in upward direction by ejecting the fuel in downward direction. Fuel mass is decreased continuously while speed of the Rocket is increases continuously
- Conveyor belt : The belt which is used to transport the material is called conveyor belt. The mass continuously deposits on the belt thereby belt gains the momentum and it moves with constant velocity.

Motion of Conveyor belt

- Consider a conveyor belt is moving with velocity V and let a material is deposited from a hopper on the belt as shown in figure below.



Power supplied by the force is = P_w =
$\vec{F} \cdot \vec{V} = \frac{dm}{dt} \cdot \vec{V} \cdot \vec{V} = \frac{dm}{dt} \cdot V^2$.
Above equation can be written as

$$P_W = \frac{d}{dt}(mV^2 + MV^2)$$

$= 2\frac{d}{dt}\left(\frac{1}{2}mV^2 + \frac{1}{2}MV^2\right) = 2\frac{dT}{dt}$. Here T is kinetic energy of the system.

Let m is the mass of the material deposited on the belt and M is the mass of the belt. If **V** is the velocity of the belt then due to material deposition the force is acted upon the belt .
Total momentum of the belt with material is
$\vec{P} = (\mathrm{M+m})\vec{V}$ (Hopper is assumed to be at rest).
$\vec{F} = \frac{d\vec{P}}{dt} = \frac{d}{dt}\left((M+m)\vec{V}\right)$. Since M and **V** is constant so we get $\vec{F} = \frac{d\vec{P}}{dt} = \frac{dm}{dt} \cdot \vec{V}$.
It shows that faster the material is deposited more is the force required to move the belt to keep moving with constant velocity.

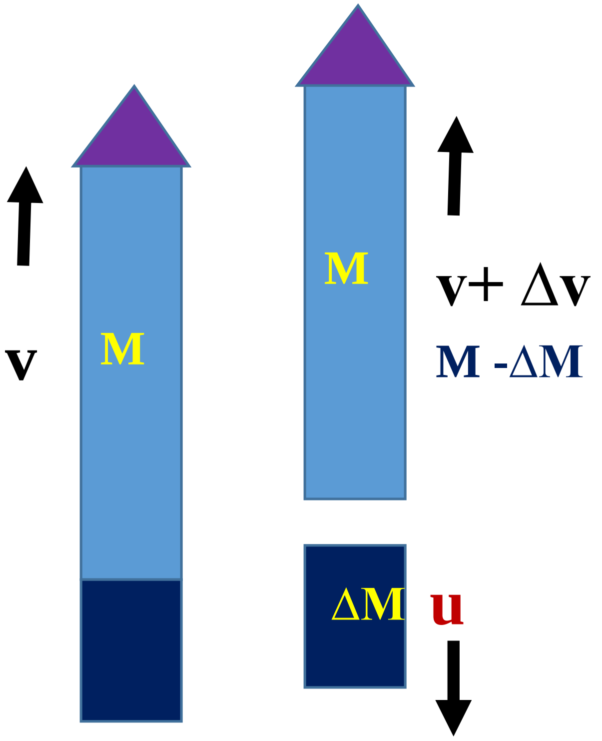
So Power supplied by the force = twice the K.E. of the system.

ROCKET MOTION

- Rocket motion is an example of motion of variable mass. Here mass of fuel is ejected , due to which mass of the rocket decreases but velocity of the rocket increases continuously till fuel gets exhausted.
- The motion of rocket is based on conservation of momentum.
- The rocket has a chamber in which fuel can be liquid or solid, is burnt.
- The large quantity of heat produced the pressure inside the chamber , resulting into ejection of burnt up gases in the form of jet.
- According to conservation of momentum, momentum lost by jet is equal to momentum gained by the rocket. So the rocket pushed in forward direction due to the jet momentum in backward direction.
- If rocket is launched from the Earth, gravitational pull can not be neglected, but at present we consider the rocket motion in space, so that gravitational pull is neglected.

Derivation for the instantaneous velocity of the rocket

- Consider a rocket whose vehicle mass is M_v and mass of the fuel M_f , so total mass of the system is $M_0 = M_v + M_f$.
- Let **v** is the velocity of the rocket at any instant of time t, **u** be the relative velocity of fuel with respect to Rocket. Its speed with respect to observer on earth is **u – v.**



so momentum of the rocket at instant of time is = Mv.

At t + Δt, the linear momentum of the rocket will be = (M-ΔM)(**v**+Δ**v**)- ΔM(**u** – **v**). According to conservation of momentum

$$Mv = (M-\Delta M)(\mathbf{v}+\Delta \mathbf{v}) - \Delta M(\mathbf{u} - \mathbf{v}) \text{------------} (1)$$

$$M\,v = M\mathbf{v} + M\,\Delta \mathbf{v} - \Delta M\mathbf{v} - \Delta M\,\Delta \mathbf{v} - \Delta M\mathbf{u} + \Delta M\,\mathbf{v}$$

$$0 = M\,\Delta \mathbf{v} - \Delta M\mathbf{u} \text{ --------} (2)$$

(neglect ΔM Δ**v as they are very small**)

Consider again the equation (2) $0 = M\,\Delta\mathbf{v} - \Delta M\mathbf{u}$. We rewrite this equation in the following form. $M\,\Delta\mathbf{v} = \Delta M\mathbf{u}$ or $M\frac{\Delta\mathbf{v}}{\Delta t} = \frac{\Delta M}{\Delta t}\boldsymbol{u}$ in Δt time.

$\frac{\Delta\mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt} = a$ which is the acceleration of the rocket and $\frac{\Delta M}{\Delta t} = -\frac{dM}{dt}$ is rate of change of mass (-ve) sign is included because mass of the rocket is decreasing.

Hence we write $M\boldsymbol{a} = -\frac{dM}{dt}\boldsymbol{u}$ --------(3) This is the equation of rocket.

The right hand side of above equation is called as Thrust of the rocket.

So Thrust $= -\frac{dM}{dt}\boldsymbol{u}$. Now to get an expression for velocity of rocket, consider

$M\frac{d\mathbf{v}}{dt} = -\frac{dM}{dt}\boldsymbol{u}$, so $d\mathbf{v} = -\frac{dM}{M}\boldsymbol{u}$ ------ (4). Integrating this equation we get

$\int_{v_0}^{v} d\,\mathbf{v} = -\boldsymbol{u}\int_{M_0}^{M}\frac{dM}{M}$ $\quad\therefore \mathbf{v} - \mathbf{v_0} = -u\ln\frac{M}{M_0}$ or $\boxed{\mathbf{v} = \mathbf{v_0} - u\ln\frac{M}{M_0}}$ --------- (5)

If in time T mass of the fuel is consumed is M_f then at time t mass of the fuel burnt is m then we must have $\frac{m}{t} = \frac{M_f}{T}$ (since $-\frac{dM}{dt} = \frac{dm}{dt} = k\ (constant)$ so

m = $\frac{t}{T}M_f \ \ for\ 0 \leq t \leq T$

So Mass of the rocket at any time t is $M = M_0 - m = M_0 - \frac{t}{T}M_f$

Hence for $0 \leq t \leq T$, Equation (5) is written as

$\mathbf{v} = \mathbf{v_0} - u ln\left(1 - \frac{t}{T}\frac{M_f}{M_0}\right)$. ------- (6)

When t =T, complete fuel is consumed/ exhausted and velocity of the rocket will be maximum i.e. v_{max} ,Mass M $\rightarrow M_\mathbf{v} = M_0 - M_f$

Eq. (6) is written as $\mathbf{v}_{\boldsymbol{max}} = \mathbf{v_0} - u ln\left(1 - \frac{M_f}{M_0}\right) = \mathbf{v_0} - u ln\left(\frac{M_0 - M_f}{M_0}\right)$

$\mathbf{v}_{\boldsymbol{max}} = \mathbf{v_0} - u ln\frac{M_\mathbf{v}}{M_0}$. ------- (7) This equation can be rewritten as

$\mathbf{v}_{\boldsymbol{max}} = \mathbf{v_0} + u ln\frac{M_0}{M_\mathbf{v}} = \mathbf{v_0} + u ln\frac{M_\mathbf{v} + \boldsymbol{M_f}}{M_\mathbf{v}} = \mathbf{v_0} + u ln\left(1 + \frac{M_\mathbf{f}}{M_\mathbf{v}}\right)$ ------- (8)

So final velocity reached by the rocket will be $\mathbf{v}_{\mathbf{max}}$ **and it is given as**

$$\mathbf{v}_{max} = \mathbf{v_0} + u ln\frac{M_\mathbf{0}}{M_\mathbf{v}}$$

where M_v will be the final mass of the rocket.

Maximum distance covered by the rocket will be integration of velocity .