(UNIVERSITY OF MUMBAI)

Syllabus for: S.Y.B.Sc./S.Y.B.A.

Program: B.Sc./B/A.
Course: Mathematics
Choice based Credit System (CBCS)
with effect from the
academic year 2018-19

SEMESTER III

CALCULUS III				
Course Code	UNIT	TOPICS	Credits	L/Week
USMT 301, UAMT 301	I	Functions of several variables	2	3
	II	Differentiation		
	III	Applications		
ALGEBRA III				
USMT 302 ,UAMT 302	I	Linear Transformations and Matrices	2	3
	II	Determinants		
	III	Inner Product Spaces		
DISCRETE MATHEMATICS				
USMT 303	I	Permutations and Recurrence Relation	2	3
	II	Preliminary Counting		
	III	Advanced Counting		
PRACTICALS				
USMTP03		Practicals based on USMT301, USMT 302 and USMT 303	3	5
UAMTP03		Practicals based on UAMT301, UAMT 302	2	4

SEMESTER IV

CALCULUS IV				
Course Code	UNIT	TOPICS	Credits	L/Week
USMT 401, UAMT 401	I	Riemann Integration	2	3
	II	Indefinite Integrals and Improper Integrals		
	III	Beta and Gamma Functions And Applications		
ALGEBRA IV				
USMT 402 ,UAMT 402	I	Groups and Subgroups	2	3
	II	Cyclic Groups and Cyclic subgroups		
	III	Lagrange's Theorem and Group Homomorphism		
ORDINARY DIFFERENTIAL EQUATIONS				
USMT 403	I	First order First degree Differential equations	2	3
	II	Second order Linear Differential equations		
	III	Linear System of Ordinary Differential Equations		
PRACTICALS				
USMTP04		Practicals based on USMT401, USMT 402 and USMT 403	3	5
UAMTP04		Practicals based on UAMT401, UAMT 402	2	4

Teaching Pattern for Semester III

1. Three lectures per week per course. Each lecture is of 48 minutes duration.
2. One Practical (2L) per week per batch for courses USMT301, USMT 302 combined and one Practical (3L) per week for course USMT303 (the batches tobe formed as prescribed by the University. Each practical session is of 48 minutes duration.)

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S.Y.B.Sc. / S.Y.B.A. Mathematics

SEMESTER III

USMT 301, UAMT 301: CALCULUS III

Note: All topics have to be covered with proof in details (unless mentioned otherwise) and examples.

Unit I: Functions of several variables (15 Lectures)

1. The Euclidean inner product on $\mathbb{R}^n$ and Euclidean norm function on $\mathbb{R}^n$, distance between two points, open ball in $\mathbb{R}^n$, definition of an open subset of $\mathbb{R}^n$, neighbourhood of a point in $\mathbb{R}^n$, sequences in $\mathbb{R}^n$, convergence of sequences- these concepts should be specifically discussed for $n = 3$ and $n = 3$.
2. Functions from $\mathbb{R}^n \longrightarrow \mathbb{R}$ (scalar fields) and from $\mathbb{R}^n \longrightarrow \mathbb{R}^m$ (vector fields), limits, continuity of functions, basic results on limits and continuity of sum, difference, scalar multiples of vector fields, continuity and components of a vector fields.
3. Directional derivatives and partial derivatives of scalar fields.
4. Mean value theorem for derivatives of scalar fields.

Reference for Unit I:
Sections 8.1, 8.2, 8.3, 8.4, 8.5, 8.6, 8.7, 8.8, 8.9, 8.10 of Calculus, Vol. 2 (Second Edition) by Apostol.

Unit II: Differentiation (15 Lectures)

1. Differentiability of a scalar field at a point of $\mathbb{R}^n$ (in terms of linear transformation) and on an open subset of $\mathbb{R}^n$, the total derivative, uniqueness of total derivative of a differentiable function at a point, simple examples of finding total derivative of functions such as $f(x, y) = x^2 + y^2, f(x, y, z) = x + y + z,$, differentiability at a point of a function f implies continuity and existence of direction derivatives of f at the point, the existence of continuous partial derivatives in a neighbourhood of a point implies differentiability at the point.

2. Gradient of a scalar field, geometric properties of gradient, level sets and tangent planes.
3. Chain rule for scalar fields.
4. Higher order partial derivatives, mixed partial derivatives, sufficient condition for equality of mixed partial derivative.

Reference for Unit II:
Sections 8.11, 8.12, 8.13, 8.14, 8.15, 8.16, 8.17, 8.23 of Calculus, Vol.2 (Second Edition) by T. Apostol, John Wiley.

Unit III: Applications (15 lectures)

1. Second order Taylor's formula for scalar fields.
2. Differentiability of vector fields, definition of differentiability of a vector field at a point, Jacobian matrix, differentiability of a vector field at a point implies continuity. The chain rule for derivative of vector fields (statements only)
3. Mean value inequality.
4. Hessian matrix, Maxima, minima and saddle points.
5. Second derivative test for extrema of functions of two variables.
6. Method of Lagrange Multipliers.

Reference for Unit III:
Sections 8.18, 8.19, 8.20, 8.21, 8.22, 9.9, 9.10, 9.11, 9.12, 9.13, 9.14 9.13, 9.14 from Apostol, Calculus Vol. 2, (Second Edition) by T. Apostol.

Recommended Text Books:

1. T. Apostol: Calculus, Vol. 2, John Wiley.
2. J. Stewart, Calculus, Brooke/ Cole Publishing Co.

Additional Reference Books

(1) G.B. Thoman and R. L. Finney, Calculus and Analytic Geometry, Ninth Edition, Addison-Wesley, 1998.

(2) Sudhir R. Ghorpade and Balmohan V. Limaye, A Course in Multivariable Calculus and Analysis, Springer International Edition.

(3) Howard Anton, Calculus- A new Horizon, Sixth Edition, John Wiley and Sons Inc, 1999.

USMT 302/UAMT 302: ALGEBRA III

Note: Revision of relevant concepts is necessary.

Unit 1: Linear Transformations and Matrices (15 lectures)

1. Review of linear transformations: Kernel and image of a linear transformation, Rank-Nullity theorem (with proof), Linear isomorphisms, inverse of a linear isomorphism, Any $n-$ dimensional real vector space is isomorphic to $\mathbb{R}^n$.

2. The matrix units, row operations, elementary matrices, elementary matrices are invertible and an invertible matrix is a product of elementary matrices.

3. Row space, column space of an $m \times n$ matrix, row rank and column rank of a matrix, Equivalence of the row and the column rank, Invariance of rank upon elementary row or column operations.

4. Equivalence of rank of an $m \times n$ matrix A and rank of the linear transformation $L_A : \mathbb{R}^n \longrightarrow \mathbb{R}^m \quad (L_A(X) = AX)$. The dimension of solution space of the system of linear equations $AX = 0$ equals $n-$rank(A).

5. The solutions of non-homogeneous systems of linear equations represented by $AX = B$, Existence of a solution when rank$(A)=$ rank(A, B), The general solution of the system is the sum of a particular solution of the system and the solution of the associated homogeneous system.
   Reference for Unit 1: Chapter VIII, Sections 1, 2 of Introduction to Linear Algebra, Serge Lang, Springer Verlag and Chapter 4, of Linear Algebra A Geometric Approach, S. Kumaresan, Prentice-Hall of India Private Limited, New Delhi.

Unit II: Determinants (15 Lectures)

1. Definition of determinant as an $n-$linear skew-symmetric function from $\mathbb{R}^n \times \mathbb{R}^n \times \ldots \times \mathbb{R}^n \longrightarrow \mathbb{R}$ such that determinant of $(E^1, E^2, \ldots, E^n)$ is 1, where E^j denotes the j^{th} column of the $n \times n$ identity matrix I_n. Determinant of a matrix as determinant of its column vectors (or row vectors). Determinant as area and volume.

2. Existence and uniqueness of determinant function via permutations, Computation of determinant of $2 \times 2, 3 \times 3$ matrices, diagonal matrices, Basic results on determinants such as $\det(A^t) = \det(A), \det(AB) = \det(A)\det(B)$, Laplace expansion of a determinant, Vandermonde determinant, determinant of upper triangular and lower triangular matrices.

3. Linear dependence and independence of vectors in $\mathbb{R}^n$ using determinants, The existence and uniqueness of the system $AX = B$, where A is an $n \times n$ matrix wither $\det(A) \neq 0$, Cofactors and minors, Adjoint of an $n \times n$ matrix A, Basic results such as $A\text{adj}(A) = \det(A)I_n$. An $n \times n$ real matrix A is invertible if and only if $\det(A) \neq 0, A^{-1} = \dfrac{1}{\det(A)}\text{adj}(A)$ for an invertible matrix A, Cramer's rule.

4. Determinant as area and volume.

References for Unit 2: Chapter VI of Linear Algebra A geometric approach, S. Kumaresan, Prentice Hall of India Private Limited, 2001 and Chapter VII Introduction to Linear Algebra, Serge Lang, Springer Verlag.

Unit III: Inner Product Spaces (15 Lectures)

1. Dot product in $\mathbb{R}^n$, Definition of general inner product on a vector space over $\mathbb{R}$. Examples of inner product including the inner product $< f, g >= \displaystyle\int_{-\pi}^{\pi} f(t)g(t)\ dt$ on $C[-\pi, \pi]$, the space of continuous real valued functions on $[-\pi, \pi]$.

2. Norm of a vector in an inner product space. Cauchy-Schwartz inequality, Triangle inequality, Orthogonality of vectors, Pythagoras theorem and geometric applications in $\mathbb{R}^2$, Projections on a line, The projection being the closest approximation, Orthogonal complements of a subspace, Orthogonal complements in $\mathbb{R}^2$ and $\mathbb{R}^3$. Orthogonal sets and orthonormal sets in an inner product space, Orthogonal and orthonormal bases. Gram-Schmidt orthogonalization process, Simple examples in $\mathbb{R}^3, \mathbb{R}^4$.

Reference of Unit 3: Chapter VI, Sections 1,2 of Introduction to Linear Algebra, Serge Lang, Springer Verlag and Chapter 5, of Linear Algebra A Geometric Approach, S. Kumaresan, Prentice-Hall of India Private Limited, New Delhi.

Recommended Books:

1. Serge Lang: Introduction to Linear Algebra, Springer Verlag.
2. S. Kumaresan: Linear Algebra A geometric approach, Prentice Hall of India Private Limited.

Additional Reference Books:

1. M. Artin: Algebra, Prentice Hall of India Private Limited.
2. K. Hoffman and R. Kunze: Linear Algebra, Tata McGraw-Hill, New Delhi.
3. Gilbert Strang: Linear Algebra and its applications, International Student Edition.
4. L. Smith: Linear Algebra, Springer Verlag.
5. A. Ramachandra Rao and P. Bhima Sankaran: Linear Algebra, Tata McGraw-Hill, New Delhi.
6. T. Banchoff and J. Wermer: Linear Algebra through Geometry, Springer Verlag Newyork, 1984.
7. Sheldon Axler: Linear Algebra done right, Springer Verlag, Newyork.
8. Klaus Janich: Linear Algebra.
9. Otto Bretcher: Linear Algebra with Applications, Pearson Education.
10. Gareth Williams: Linear Algebra with Applications, Narosa Publication.

USMT 303: Discrete Mathematics

Unit I: Permutations and Recurrence relation (15 lectures)

1. Permutation of objects, S_n, composition of permutations, results such as every permutation is a product of disjoint cycles, every cycle is a product of transpositions, even and odd permutation, rank and signature of a permutation, cardinality of S_n, A_n
2. Recurrence Relations, definition of non-homogeneous, non-homogeneous, linear , non-linear recurrence relation, obtaining recurrence relation in counting problems, solving homogeneous as well as non homogeneous recurrence relations by using iterative methods, solving a homogeneous recurrence relation of second degree using algebraic method proving the necessary result.

Recommended Books:

1. Norman Biggs: Discrete Mathematics, Oxford University Press.
2. Richard Brualdi: Introductory Combinatorics, John Wiley and sons.
3. V. Krishnamurthy: Combinatorics-Theory and Applications, Affiliated East West Press.
4. Discrete Mathematics and its Applications, Tata McGraw Hills.
5. Schaum's outline series: Discrete mathematics,
6. Applied Combinatorics: Allen Tucker, John Wiley and Sons.

Unit II: Preliminary Counting (15 Lectures)

1. Finite and infinite sets, countable and uncountable sets examples such as $\mathbb{N}, \mathbb{Z}, \mathbb{N} \times \mathbb{N}, \mathbb{Q}$,$(0,1), \mathbb{R}$
2. Addition and multiplication Principle, counting sets of pairs, two ways counting.
3. Stirling numbers of second kind. Simple recursion formulae satisfied by $S(n,k)$ for $k = 1, 2, \cdots, n-1, n$
4. Pigeonhole principle and its strong form, its applications to geometry, monotonic sequences etc.

Unit III: Advanced Counting (15 Lectures)

1. Binomial and Multinomial Theorem, Pascal identity, examples of standard identities such as the following with emphasis on combinatorial proofs.
   - $\sum_{k=0}^{r} \binom{m}{k}\binom{n}{r-k} = \binom{m+n}{r}$
   - $\sum_{i=r}^{n} \binom{i}{r} = \binom{n+1}{r+1}$
   - $\sum_{i=0}^{k} \binom{k}{i}^2 = \binom{2k}{k}$
   - $\sum_{i=0}^{n} \binom{n}{i} = 2^n$
2. Permutation and combination of sets and multi-sets, circular permutations, emphasis on solving problems.
3. Non-negative and positive solutions of equation $x_1 + x_2 + \cdots + x_k = n$
4. Principal of inclusion and exclusion, its applications, derangements, explicit formula for d_n, deriving formula for Euler's function $\phi(n)$.

USMT P03/UAMTP03 Practicals

Suggested Practicals for USMT 301/UAMT303

1. Sequences in $\mathbb{R}^2$ and $\mathbb{R}^3$, limits and continuity of scalar fields and vector fields, using "definition and otherwise" , iterated limits.
2. Computing directional derivatives, partial derivatives and mean value theorem of scalar fields.
3. Total derivative, gradient, level sets and tangent planes.
4. Chain rule, higher order derivatives and mixed partial derivatives of scalar fields.
5. Taylor's formula, differentiation of a vector field at a point, finding Hessian/Jacobean matrix, Mean Value Inequality.
6. Finding maxima, minima and saddle points, second derivative test for extrema of functions of two variables and method of Lagrange multipliers.
7. Miscellaneous Theoretical Questions based on full paper

Suggested Practicals for USMT302/UAMT302:

1. Rank-Nullity Theorem.
2. System of linear equations.
3. Determinants , calculating determinants of 2×2 matrices, $n \times n$ diagonal, upper triangular matrices using definition and Laplace expansion.
4. Finding inverses of $n \times n$ matrices using adjoint.
5. Inner product spaces, examples. Orthogonal complements in $\mathbb{R}^2$ and $\mathbb{R}^3$.
6. Gram-Schmidt method.
7. Miscellaneous Theoretical Questions based on full paper

Suggested Practicals for USMT 303:

1. Derangement and rank signature of permutation.
2. Recurrence relation.
3. Problems based on counting principles, Two way counting.
4. Stirling numbers of second kind, Pigeon hole principle.
5. Multinomial theorem, identities, permutation and combination of multi-set.
6. Inclusion-Exclusion principle. Euler phi function.
7. Miscellaneous theory quesitons from all units.

SEMESTER IV

USMT 401/UAMT 401: CALCULUS IV

Note: All topics have to be covered with proof in details (unless mentioned otherwise) and examples.

Unit I: Riemann Integration (15 Lectures)

Approximation of area, Upper/Lower Riemann sums and properties, Upper/Lower integrals, Definition of Riemann integral on a closed and bounded interval, Criterion of Riemann integrability, if $a < c < b$ then $f \in R[a, b]$, if and only if $f \in R[a, c]$ and $f \in R[c, b]$ and $\int_a^b f = \int_a^c f + \int_c^b f$.

Properties:

(i) $f, g \in R[a, b] \Longrightarrow f + g, \lambda f \in R[a, b]$.

(ii) $\int_a^b (f + g) = \int_a^b f + \int_a^b g$.

(iii) $\int_a^b \lambda f = \lambda \int_a^b f$.

(iv) $f \in R[a, b] \Longrightarrow |f| \in R[a, b]$ and $|\int_a^b f| \leq \int_a^b |f|$,

(v) $f \geq 0, f \in C[a, b] \Longrightarrow f \in R[a, b]$.

(vi) If f is bounded with finite number of discontinuities then $f \in R[a, b]$, generalize this if f is monotone then $f \in R[a, b]$.

Unit II: Indefinite and improper integrals (15 lectures)

Continuity of $F(x) = \int_a^x f(t)\ dt$ where $f \in R[a, b]$, Fundamental theorem of calculus, Mean value theorem, Integration by parts, Leibnitz rule, Improper integrals-type 1 and type 2, Absolute convergence of improper integrals, Comparison tests, Abel's and Dirichlet's tests.

Unit III: Applications (15 lectures)

(1) β and Γ functions and their properties, relationship between β and Γ functions (without proff).

(2) Applications of definite Integras: Area between curves, finding volumes by sicing, volumes of solids of revolution-Disks and Washers, Cylindrical Shells, Lengths of plane curves, Areas of surfaces of revolution.

References:

(1) Calculus Thomas Finney, ninth edition section 5.1, 5.2, 5.3, 5.4, 5.5, 5.6.

(2) R. R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.

(3) Ajit Kumar, S.Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014.
(4) T. Apostol, Calculus Vol.2, John Wiley.
(5) K. Stewart, Calculus, Booke/Cole Publishing Co, 1994.
(6) J. E. Marsden, A.J. Tromba and A. Weinstein, Basic multivariable calculus.
(7) Bartle and Sherbet, Real analysis.

USMT 402/ UAMT 402: ALGEBRA IV

Unit I: Groups and Subgroups (15 Lectures)

(a) Definition of a group, abelian group, order of a group, finite and infinite groups. Examples of groups including:

- i) $\mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}$ under addition.
- ii) $\mathbb{Q}^*(= \mathbb{Q} \setminus \{0\}), \mathbb{R}^*(= \mathbb{R} \setminus \{0\}), \mathbb{C}^*(= \mathbb{C} \setminus \{0\}).\mathbb{Q}^+$(= positive rational numbers) under multiplication.
- iii) $\mathbb{Z}_n$, the set of residue classes modulo n under addition.
- iv) $U(n)$, the group of prime residue classes modulo n under multiplication.
- v) The symmetric group S_n.
- vi) The group of symmetries of a plane figure. The Dihedral group D_n as the group of symmetries of a regular polygon of n sides (for $n = 3, 4$).
- vii) Klein 4-group.
- viii) Matrix groups $M_{n\times n}(\mathbb{R})$ under addition of matrices, $GL_n(\mathbb{R})$, the set of invertible real matrices, under multiplication of matrices.
- ix) Examples such as S^1 as subgroup of C, μ_n the subgroup of $n-$th roots of unity.

(b) Properties such as

1) In a group $(G, .)$ the following indices rules are true for all integers n, m.
   - i) $a^n a^m = a^{n+m}$ for all a in G.
   - ii) $(a^n)^m = a^{nm}$ for all a in G.
   - iii) $(ab)^n = a^n b^n$ for all ab in G whenever $ab = ba$.
2) In a group $(G, .)$ the following are true:
   - i) The identity element e of G is unique.
   - ii) The inverse of every element in G is unique.
   - iii) $(a^{-1})^{-1} = a$ for all a in G.
   - iv) $(a.b)^{-1} = b^{-1}a^{-1}$ for all a, b in G.
   - v) If $a^2 = e$ for every a in G then $(G, .)$ is an abelian group.
   - vi) $(aba^{-1})^n = ab^n a^{-1}$ for every a, b in G and for every integer n.
   - vii) If $(a.b)^2 = a^2.b^2$ for every a, b in G then $(G, .)$ is an abelian group.
   - viii) $(\mathbb{Z}_n^*, .)$ is a group if and only if n is a prime.
3) Properties of order of an element such as: (n and m are integers.)
   - i) If $o(a) = n$ then $a^m = e$ if and only if n/m.
   - ii) If $o(a) = nm$ then $o(a^n) = m$.
   - iii) If $o(a) = n$ then $o(a^m) = \dfrac{n}{(n, m)}$,. where (n, m) is the GCD of n and m.

- iv) $o(aba^{-1}) = o(b)$ and $o(ab) = o(ba)$.
- v) If $o(a) = m$ and $o(b) = m, ab = ba, (n, m) = 1$ then $o(ab) = nm$.

(c) Subgroups

- i) Definition, necessary and sufficient condition for a non-empty set to be a Subgroup.
- ii) The center $Z(G)$ of a group is a subgroup.
- iii) Intersection of two (or a family of) subgroups is a subgroup.
- iv) Union of two subgroups is not a subgroup in general. Union of two subgroups is a subgroup if and only if one is contained in the other.
- v) If H and K are subgroups of a group G then HK is a subgroup of G if and only if $HK = KH$.

Reference for Unit I:

(1) I.N. Herstein, Topics in Algebra.

(2) P.B. Bhattacharya, S.K. Jain, S. Nagpaul. Abstract Algebra.

Unit II: Cyclic groups and cyclic subgroups (15 Lectures)

(a) Cyclic subgroup of a group, cyclic groups, (examples including $\mathbb{Z}, \mathbb{Z}_n$ and μ_n).

(b) Properties such as:

- (i) Every cyclic group is abelian.
- (ii) Finite cyclic groups, infinite cyclic groups and their generators.
- (iii) A finite cyclic group has a unique subgroup for each divisor of the order of the group.
- (iv) Subgroup of a cyclic group is cyclic.
- (v) In a finite group $G, G =< a >$ if and only if $o(G) = o(a)$.
- (vi) If $G =< a >$ and $o(a) = n$ then $G =< a^m >$ if and only if $(n, m) = 1$.
- (vii) If G is a cyclic group of order p^n and $H < G, K < G$ then prove that either $H \subseteq K$ or $K \subseteq H$.

References for Unit II:

(1) I.N. Herstein, Topics in Algebra.

(2) P.B. Bhattacharya, S.K. Jain, S. Nagpaul. Abstract Algebra.

Unit III: Lagrange's Theorem and Group homomorphism (15 Lectures)

(a) Definition of Coset and properties such as :

1) IF H is a subgroup of a group G and $x \in G$ then
   - (i) $xH = H$ if and only if $x \in H$.
   - (ii) $Hx = H$ if and only if $x \in H$.
2) If H is a subgroup of a group G and $x, y \in G$ then
   - (i) $xH = yH$ if and only if $x^{-1}y \in H$.
   - (ii) $Hx = Hy$ if and only if $xy^{-1} \in H$.

3) Lagrange's theorem and consequences such as Fermat's Little theorem, Euler's theorem and if a group G has no nontrivial subgroups then order of G is a prime and G is Cyclic.

(b) Group homomorphisms and isomorphisms, automorphisms

i) Definition.

ii) Kernel and image of a group homomorphism.

iii) Examples including inner automorphism.

Properties such as:

(1) $f : G \longrightarrow G'$ is a group homomorphism then $\ker f < G$.

(2) $f : G \longrightarrow G'$ is a group homomorphism then $\ker f = \{e\}$ if and only if f is 1-1.

(3) $f : G \longrightarrow G'$ is a group homomorphism then

(i) G is abelian if and only if G' is abelian.

(ii) G is cyclic if and only if G' is cyclic.

Reference for Unit III:

1. I.N. Herstein, Topics in Algebra.
2. P.B. Bhattacharya, S.K. Jain, S. Nagpaul. Abstract Algebra.

Recommended Books:

1. I.N. Herstein, Topics in Algebra, Wiley Eastern Limied, Second edition.
2. N.S. Gopalkrishnan, University Algebra, Wiley Eastern Limited.
3. M. Artin, Algebra, Prentice Hall of India, New Delhi.
4. P.B. Bhattacharya, S.K. Jain, S. Nagpaul. Abstract Algebra, Second edition, Foundation Books, New Delhi, 1995.
5. J.B. Fraleigh, A first course in Abstract Algebra, Third edition, Narosa, New Delhi.
6. J. Gallian. Contemporary Abstract Algebra. Narosa, New Delhi.
7. COmbinatroial Techniques by Sharad S. Sane, Hindustan Book Agency.

Additional Reference Books:

1. S. Adhikari. An introduction to Commutative Algebra and Number theory. Narosa Publishing House.
2. T. W. Hungerford. Algebra, Springer.
3. D. Dummit, R. Foote. Abstract Algebra, John Wiley & Sons, Inc.
4. I.S. Luther, I.B.S. Passi. Algebra. Vol. I and II.

USMT 403: ORDINARY DIFFERENTIAL EQUATIONS

Unit I: First order First degree Differential equations (15 Lectures)

(1) Definition of a differential equation, order, degree, ordinary differential equation and partial differential equation, linear and non linear ODE.

(2) Existence and Uniqueness Theorem for the solution of a second order initial value problem (statement only), Definition of Lipschitz function, Examples based on verifying the conditions of existence and uniqueness theorem

(3) Review of Solution of homogeneous and non-homogeneous differential equations of first order and first degree. Notion of partial derivatives. Exact Equations: General solution of Exact equations of first order and first degree. Necessary and sufficient condition for $Mdx + Ndy = 0$ to be exact. Non-exact equations: Rules for finding integrating factors (without proof) for non exact equations, such as :

i) $\dfrac{1}{M\ x + N\ y}$ is an I.F. if $M\ x + N\ y \neq 0$ and $Mdx + Ndy = 0$ is homogeneous.

ii) $\dfrac{1}{M\ x - N\ y}$ is an I.F. if $M\ x - N\ y \neq 0$ and $Mdx + Ndy = 0$ is of the form $f_1(x, y)\ y\ dx + f_2(x, y)\ x\ dy = 0$.

iii) $e^{\int f(x)\ dx}$ (resp $e^{\int g(y)\ dy}$) is an I.F. if $N \neq 0$ (resp $M \neq 0$) and $\dfrac{1}{N}\left(\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}\right)$ $\left(\text{resp}\dfrac{1}{M}\left(\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}\right)\right)$ is a function of x (resp y) alone, say $f(x)$ (resp $g(y)$).

iv) Linear and reducible linear equations of first order, finding solutions of first order differential equations of the type for applications to orthogonal trajectories, population growth, and finding the current at a given time.

Unit II: Second order Linear Differential equations (15 Lectures)

1. Homogeneous and non-homogeneous second order linear differentiable equations: The space of solutions of the homogeneous equation as a vector space. Wronskian and linear independence of the solutions. The general solution of homogeneous differential equations. The general solution of a non-homogeneous second order equation. Complementary functions and particular integrals.

2. The homogeneous equation with constant coefficients. auxiliary equation. The general solution corresponding to real and distinct roots, real and equal roots and complex roots of the auxiliary equation.

3. Non-homogeneous equations: The method of undetermined coefficients. The method of variation of parameters.

Unit III: Linear System of ODEs (15 Lectures)

Existence and uniqueness theorems to be stated clearly when needed in the sequel. Study of homogeneous linear system of ODEs in two variables: Let $a_1(t), a_2(t), b_1(t), b_2(t)$ be continuous real valued functions defined on $[a, b]$. Fix $t_0 \in [a, b]$. Then there exists a unique solution $x = x(t), y = y(t)$ valid throughout $[a, b]$ of the following system:

$$\frac{dx}{dt} = a_1(t)x + b_1(t)y,$$
$$\frac{dy}{dt} = a_2(t)x + b_2(t)y$$

satisfying the initial conditions $x(t_0) = x_0 \& y(t_0) = y_0$.
The Wronskian $W(t)$ of two solutions of a homogeneous linear system of ODEs in two variables, result: W (t) is identically zero or nowhere zero on [a, b]. Two linearly independent solutions and the general solution of a homogeneous linear system of ODEs in two variables.
Explicit solutions of Homogeneous linear systems with constant coefficients in two variables, examples.

Recommended Text Books for Unit I and II:

1. G. F. Simmons, Differential equations with applications and historical notes, McGraw Hill.

2. E. A. Coddington, An introduction to ordinary differential equations, Dover Books.

Recommended Text Book for Unit III:
G. F. Simmons, Differential equations with applications and historical notes, McGraw Hill.

USMT P04/UAMT P04 Practicals.

Suggested Practicals for USMT401/UAMT401:

1. Calculation of upper sum, lower sum and Riemann integral.
2. Problems on properties of Riemann integral.
3. Problems on fundamental theorem of calculus, mean value theorems, integration by parts, Leibnitz rule.
4. Convergence of improper integrals, applications of comparison tests, Abel's and Dirichlet's tests, and functions.
5. Beta Gamma Functions
6. Problems on area, volume, length.
7. Miscellaneous Theoretical Questions based on full paper.

Suggested Practicals for USMT402/UAMT 402:

1. Examples and properties of groups.
2. Group of symmetry of equilateral triangle, rectangle, square.
3. Subgroups.
4. Cyclic groups, cyclic subgroups, finding generators of every subgroup of a cyclic group.
5. Left and right cosets of a subgroup, Lagrange's Theorem.

6. Group homomorphisms, isomorphisms.
7. Miscellaneous Theoretical questions based on full paper.

Suggested Practicals for USMT403:

1. Solving exact and non exact equations.
2. Linear and reducible to linear equations, applications to orthogonal trajectories, population growth, and finding the current at a given time.
3. Finding general solution of homogeneous and non-homogeneous equations, use of known solutions to find the general solution of homogeneous equations.
4. Solving equations using method of undetermined coefficients and method of variation of parameters.
5. Solving second order linear ODEs
6. Solving a system of first order linear ODES.
7. Miscellaneous Theoretical questions from all units.

Scheme of Examination

I. **Semester End Theory Examinations:** There will be a Semester-end external Theory examination of 100 marks for each of the courses USMT301/UAMT301, USMT302/UAMT302, USMT303 of Semester III and USMT401/UAMT401, USMT402/UAMT402, USMT403 of semester IV to be conducted by the University.

   1. Duration: The examinations shall be of 3 Hours duration.
   2. Theory Question Paper Pattern:
      a) There shall be FIVE questions. The first question Q1 shall be of objective type for 20 marks based on the entire syllabus. The next three questions Q2, Q2, Q3 shall be of 20 marks, each based on the units I, II, III respectively. The fifth question Q5 shall be of 20 marks based on the entire syllabus.
      b) All the questions shall be compulsory. The questions Q2, Q3, Q4, Q5 shall have internal choices within the questions. Including the choices, the marks for each question shall be 30-32.
      c) The questions Q2, Q3, Q4, Q5 may be subdivided into sub-questions as a, b, c, d & e, etc and the allocation of marks depends on the weightage of the topic.
      d) The question Q1 may be subdivided into 10 sub-questions of 2 marks each.

II. **Semester End Examinations Practicals:**
At the end of the Semesters III and IV, Practical examinations of three hours duration and 150 marks shall be conducted for the courses USMTP03, USMTP04.

At the end of the Semesters III and IV, Practical examinations of three hours duration and 150 marks shall be conducted for the courses UAMTP03, UAMTP04.

In semester III, the Practical examinations for USMT301/UAMT301 and USMT302/UAMT302 are held together by the college. The Practical examination for USMT303 is held **separately** by the college.

In semester IV, the Practical examinations for USMT401/UAMT401 and USMT402/UAMT402 are held together by the college. The Practical examination for USMT403 is held **separately** by the college.

Paper pattern: The question paper shall have three parts A, B, C.
Each part shall have two Sections.

Section I Objective in nature: Attempt any Eight out of Twelve multiple choice questions. ($8 \times 3 = 24$ Marks)

Section II Problems: Attempt any Two out of Three. ($8 \times 2 = 16$ Marks)

Practical Course	Part A	Part B	**Part C**	Marks out of	duration
USMTP03	Questions from USMT301	Questions from USMT302	Questions from USMT303	120	3 hours
UAMTP03	Questions from UAMT301	Questions from UAMT302	—	80	2 hours
USMTP04	Questions from USMT401	Questions from USMT402	Questions from USMT403	120	3 hours
UAMTP03	Questions from UAMT401	Questions from UAMT402	—	80	2 hours

Marks for Journals and Viva:
For each course USMT301/UAMT301, USMT302/UAMT302, USMT303, USMT401/UAMT401, USMT402/UAMT402 and USMT403:

1. Journals: 5 marks.
2. Viva: 5 marks.

Each Practical of every course of Semester III and IV shall contain 10 (ten) problems out of which minimum 05 (five) have to be written in the journal. A student must have a certified journal before appearing for the practical examination.