TYBSC (MATHS), PAPER-III, METRIC SPACES(CONTINUOUS FUNCTIONS) QUESTION BANKS

	Choose correct alternative in each of the following			
1	**Let (X, d_1) and (Y, d_2) be two metric spaces, then $f: X \to Y$ is continuous at $a \in X$, if for given $\epsilon > 0, \exists\ \delta > 0$ such that**			
	(a)	$d_1(x,a) > \delta \Rightarrow d_2(f(x), f(a)) < \epsilon$	**(b)**	$d_1(x,a) < \delta \Rightarrow d_2(f(x), f(a)) < \epsilon$
	(c)	$d_1(x,a) < \delta \Rightarrow d_2(f(x), f(a)) > \epsilon$	**(d)**	$d_1(x,a) > \delta \Rightarrow d_2(f(x), f(a)) > \epsilon$
2	**If (X, d) is a metric space, then identity function function $f: X \to X$ is**			
	(a)	**Continuous function.**	**(b)**	**Continuous only if X is finite.**
	(c)	**Continuous only if X is countable**	**(d)**	**Discontinues on X**
3	**Let $f, g: X \to Y$ be two continuous functions, α be any scalar then**			
	(a)	$\frac{f}{g}$ **is continuous**	**(b)**	$\frac{g}{f}$ **is continuous**
	(c)	$\alpha\ f$ **is continuous for $\alpha > 0$**	**(d)**	αf **is continuous.**
4	**Let $f, g: R^n \to R$ be continuous functions, such that $h(x) = \{f(x), g(x)\}$ and $g(x) = \{\ f(x), g(x)\}$ then**			
	(a)	**$h(x)$ is continuous but $g(x)$ is not continuous.**	**(b)**	**$h(x)$ is not continuous but $g(x)$ is continuous.**
	(c)	**$h(x)$ and $g(x)$ are continuous.**	**(d)**	**$h(x)$ and $g(x)$ not continuous.**
5	**Let (X, d) and (Y, ρ) be metric space, $f: X \to Y$, then which of the following statement True?**			
	(a)	**f is continuous iff f is sequentially continuous**	**(b)**	**f may be continuous but not sequentially continuous.**
	(c)	**f may not be continuous but sequentially continuous.**	**(d)**	**f is continuous iff both X and Y are closed.**
6	**Let (X, d) be a metric space and $A \subset X$. Let $f(x) = d(x, A)$ for $x \in X$, then $f: X \to R$ is**			
	(a)	**Uniformly continuous**	**(b)**	**Continuous but not uniformly continuous**
	(c)	**Not continuous**	**(d)**	**Neither continuous not uniformly continuous,**
7	**Let $A = \left\{x \in R: cos\ cos\ x = \frac{\sqrt{3}}{2}\right\}$, the distance R is usual then.**			

	(a)	A **is finite closed set**	**(b)**	**A is infinite closed set**
	(c)	**A is open set**	**(d)**	**A is bounded**
8	$f, g: R \to R$ **are any maps, such that** fog **and** gof **are continuous (distance is usual) then,**			
	(a)	$fog = gof$	**(b)**	**At least one of** f **and g is continuous**
	(c)	**Both** f **and** g **are continuous**	**(d)**	**Neither** f **nor** g **may be continuous.**
9	**Let** $X = M_2(R)$ **and** $\|A\| = \sqrt{\Sigma_{1 \le i \le j \le 2} a_{ij}^2}$ **,** $f: X \to R$ **(usual distance) defined by** $f(A) = det\ det\ A$ **then**			
	(a)	$(GL)_2(R)$ **is closed subset of** X	**(b)**	$(SL)_2(R)$ **is open subset of** X
	(c)	f **is continuous**	**(d)**	f **is not continuous.**
10	**Let** (X, d) **be compact metric space,** $f: X \to (0, \infty)$ **be continuous then** $\exists\ \epsilon > 0$ **such that**			
	(a)	$f(x) > \epsilon, \forall x \in X$	**(b)**	$f(x) < \epsilon, \forall x \in X$
	(c)	$f(x) \geq \epsilon, \forall x \in X$	**(d)**	$f(x) \leq \epsilon, \forall x \in X$
11	**Let** (X, d) **and** (Y, d_1) **be metric space, then any Lipschitz function** $f: (X, d) \to (Y, d_1)$ **is**			
	(a)	**Continuous but not uniformly continuous**	**(b)**	**Uniformly continuous**
	(c)	**Not uniformly continuous**	**(d)**	**discontinuous**
12	**A point** $x \in X$ **is called fixed point of the mapping** $T: X \to X$ **if**			
	(a)	$T(x) = x$	**(b)**	$T(x) > x$
	(c)	$T(x) < x$	**(d)**	$T(x) = 0$
13	**Let** $f: X \to Y$ **is such that** f **is continuous and** $f(x_n) \to f(x)$ **then**			
	(a)	x_n **does not converges to** x	**(b)**	$x_n \to x$
	(c)	(x_n) **may not be convergent**	**(d)**	$x_n \to x$ **for finite set** X **only**
14	**Let** (X, d) **and** (Y, d) **be two metric spaces**$f: X \to Y$ **is continuous then for each**			
	(a)	**Open subset of** G **of** X**,** $f(G)$ **is open in** Y**.**	**(b)**	**Closed subset of** H **of** X**,** $f(H)$ **is open in** Y
	(c)	**Open subset of** G **of** Y**,** $f^{-1}(G)$ **is open in** Y**.**	**(d)**	**Open subset of** G **of** Y**,** $f^{-1}(G)$ **is closed in** Y**.**
15	**Let** (X, d) **and** (Y, d_1) **be two metric spaces. and** $f: X \to Y$ **is continuous for each** $B \subseteq X$ **then**			
	(a)	$f^{-1}(\underline{B}) \subseteq \underline{f^{-1}(B)}$	**(b)**	$f(\underline{B}) \subseteq \underline{f(B)}$
	(c)	$\underline{f(B)} \subseteq f(\underline{B})$	**(d)**	$f(\underline{B}) = \underline{f(B)}$

16	**Let** (X, d) **be a complete metric space and** $T: X \to X$ **be such that** $d\big(T(x), t(y)\big) < d(x, y)$ **then**			
	(a)	T **does not have a unique point**	**(b)**	T **has two fixed points**
	(c)	T **has multiple fixed point**	**(d)**	T **has unique fixed points.**
17	**Let** (X, d) **and** (Y, d_1) **be two metric spaces. and** $f: X \to Y$ **is continuous, if** (X, d) **is compact metric space then** $f: X \to Y$ **is**			
	(a)	**Uniformly continuous**	**(b)**	**discontinuous**
	(c)	**Continuous but not uniformly**	**(d)**	**May or may not be uniformly continuous.**
18	**Let** (X, d) **and** (Y, ρ) **be metric spaces,** Y **is discrete metric space and** $f: X \to Y$ **is such that** $\rho\big(f(x), f(y)\big) \le c\, d(x, y), c \in [0,1]$ **Then for** $x \in B\left(x_0, \frac{1}{c}\right)$ **,** $x_0 \in X$			
	(a)	f **is identify function**	**(b)**	$f(x) = 0$
	(c)	$f(x) = 1$	**(d)**	f **is constant function**
19	**Let** $f: [2,3] \to [2,3]$ **be differential function and** $\lvert f'(x) \rvert < 0.5$**, then** f **is contraction on**			
	(a)	$[2,3]$	**(b)**	$[2.5, 3]$ **only**
	(c)	$[2,2.5]$ **only**	**(d)**	f **is not contraction.**
20	**Let** (X, d) and (Y, ρ) **be metric spaces,** $f, g: X \to Y$ **be continuous functions then the set** $U = \{x \in X: f(x) \neq g(x)\}$ **then**			
	(a)	U **is closed subset of** X	**(b)**	U **is open subset of** X
	(c)	U **is both open and closed subset of** X	**(d)**	U **is neither open nor closed subset of** X
21	**Let** (X, d) and (Y, ρ) **be metric spaces,** $f, g: X \to Y$ **be continuous functions. Let** U **be any open subset of** X and V **be any open subset of** Y **then**			
	(a)	$\left(f^{-1}(V)\right)^{\circ} \subseteq f^{-1}(V^{\circ})$	**(b)**	$\left(f^{-1}(U)\right)^{\circ} \subseteq f^{-1}(U^{\circ})$
	(c)	$f^{-1}(V^{\circ}) \subseteq \left(f^{-1}(V)\right)^{\circ}$	**(d)**	$f^{-1}(U^{\circ}) \subseteq \left(f^{-1}(U)\right)^{\circ}$
22	**Let** (Z, d) **and** (Y, d_1) **be metric spaces such that** $f: (Z, d) \to (X, d_1)$ **be continuous where** d **is usual distance on** N **then metric** d_1 **is**			
	(a)	**Usual only**	**(b)**	**Euclidian only**
	(c)	**Discrete only**	**(d)**	**Any metric**
23	**Let metrics** d_1 **and** d_2 **are equivalent metrics then identity map** i **on** X **defined as** $i: (X, d_1) \to (X, d_2)$ **and** $i: (X, d_2) \to (X, d_1)$ **such that**			

	(a)	**Both identity maps are discontinuous**	**(b)**	**Both identity maps are continuous**
	(c)	**Both identity maps are continuous for usual distance only**	**(d)**	**One metric is usual and other discrete only**
24	**Let** (X, d) **be a metric space and** $A \subseteq X$ **,** $f: X \to R$ **is continuous for** $f_A(x) = d(x, A)$ **where** d **is usual in** R and $f_A(x) = 0$ **then**			
	(a)	$x \in A$	**(b)**	$x \in A^{\circ}$
	(c)	$x \in closure(A)$	**(d)**	A **is singleton set**
25	**Let** d **denotes usual distance in** R **and** d_1 **denotes the discrete metric on** R **and** d_1 **denotes the discrete metric on** R **. let** $i: (R, d_1) \to (R, d)$ **be the identity map. Then**			
	(a)	$\underline{iQ} \subseteq i(\underline{Q})$	**(b)**	$\underline{i^{-1}(Q)} \subseteq i^{-1}\left(\underline{Q}\right)$
	(c)	$i^{-1}\left(\underline{Q}\right) \subseteq \underline{i^{-1}(Q)}$	**(d)**	$\underline{iQ} = i(\underline{Q})$
26	**Let** (X, d) **be a finite metric space, if** $f, g \in C(X, R)$**, then**			
	(a)	$f + g \in C(X, R)$**, but** fg **may not be in** $C(X, R)$.	**(b)**	$f + g, f - g \in C(X, R)$ **, but** $2f$ **may not be in** $C(X, R)$.
	(c)	$f + g, f - g$ and $2f \in C(X, R)$	**(d)**	$f + g, f - g \in C(X, R)$ **, but** fg **may not be in** $C(X, R)$.
27	**Let** $f: [0, 2\pi] \to R$**, such that** $f(t) = (cos\ cos\ t\ , sin\ sin\ t\ )$			
	(a)	f **is continuous**	**(b)**	f **is discontinuous**
	(c)	f **is discontinuous at** 0.	**(d)**	f **is discontinuous at 1**.
28	$f: [a, b] \to R$ **, defined as** $f(x) = x^2$ **is uniformly continuous, where**			
	(a)	$b > 0$	**(b)**	$b < 0$
	(c)	$b = 0$	**(d)**	$b \in Z$
29	**Let** (X, d) **and** (Y, d_1)**be metric spaces and such that** d **is discrete and** $f: X \to Y$ **is continuous, metric** d_1 **is**			
	(a)	**Any metric**	**(b)**	**also discrete**
	(c)	**not discrete**	**(d)**	**usual only.**
30	**Let** $f: R \to R$ **defined as** $f(x) = 3x - 2$ **then fixed point of** f **is**			
	(a)	**0**	**(b)**	**1**
	(c)	**2**	**(d)**	**3**
31	**Let** $X_1 = [0,1]; Y = [0, \infty); X_2 = (0,1) \cup (2,3), Y_2 = (0,1), X_3 = (0,1), Y_3 = \{0,1\}$**. Then there exists a continuous onto function from** $X_i \to Y_i$ **when**			
	(a)	$i = 2$	**(b)**	$i = 3$

	(c)	$i = 1,2$	**(d)**	$i = 1,2,3$
32	**Let (X, d) be a metric space where X is finite set and (Y, d') be any metric space. Let $f: X \to Y$. Then false statement is**			
	(a)	f **is continuous on** X	**(b)**	$f(X)$ **is bounded**
	(c)	**If A is open in** X, $f(A)$ **is open in Y**	**(d)**	**If B is closed in** Y, $f^{-1}(B)$ **is closed in X**
33	**Let (X, d) and (Y, d_1) be metric spaces, such that X is finite and $f: X \to Y$ is continuous, then**			
	(a)	Y **is also finite**	**(a)**	Y **is infinite**
	(c)	Y **is any metric space**	**(c)**	$d = d_1$
34	**Let (X, d) and (Y, ρ) be metric spaces, $f: X \to Y$ is continuous at $x \in X$. Let U is any open subset of Y containing $f(x)$, then there is an open set $V \subset X$, such that**			
	(a)	$f(V) \subseteq U$ **,** $x \notin X$	**(b)**	$f(V) \subseteq U$ **,** $x \in X$
	(c)	$f(V) \supseteq U, x \notin X$	**(d)**	$f(V) \supseteq U, x \in X$
35	**Let $f, g: (0,1) \to R$ such that $f(x) = \frac{1}{x}$, $g(x) = sin\ sin\ \left(\frac{1}{x}\right)$ then**			
	(a)	f **and** g **are uniformly continuous**	**(b)**	**Only one of them is uniformly continuous**
	(c)	$f.g$ **is uniformly continuous.**	**(d)**	f **and** g **are not uniformly continuous.**
36	**Let $f, g: R \to R$ such that $f(x) = x$ and $g(x) = x^2$ then**			
	(a)	f **and** g **are uniformly continuous**	**(b)**	f **and** g **are not uniformly continuous**
	(c)	**only** f **is uniformly continuous**	**(d)**	**only** g **is uniformly continuous**
37	$f(x) = \frac{1}{1+x^2}$ **is uniformly continuous on R then**			
	(a)	$x > 0$	**(b)**	$x < 0$
	(c)	$x \in Q$	**(d)**	$x \in R$
38	**Let (X, d) be a compact metric space and $f: X \to (0, \infty).(usual\ distance)$ be a continuous function. If $inf\ inf\ \{f(x): x \in X\} = m$ then**			
	(a)	m **may be zero**	**(b)**	$m > 0$
	(c)	$m = 0$	**(d)**	m **may be negative**
39	**Let (N, d) and (Y, ρ) be metric spaces such that $f: (N, d) \to (X, \rho)$ be continuous where d is usual distance on N then metric ρ is**			
	(a)	**Usual only**	**(b)**	**Euclidian only**
	(c)	**Discrete only**	**(d)**	**Any metric**
40	**Let $(X, d)\ and\ (Y, \rho)$ be metric spaces, $f, g: X \to Y$ be continuous functions such that,**			

	$H = \{x \in X: f(x) = g(x)\}$ **, let** D **be dense subset of** x **then**				
	(a)	$f(x) = 0 = g(x), \forall x \in D$	**(b)**	$f(x) < g(x), \forall x \in D$	
	(c)	$f(x) > g(x), \forall x \in D$	**(d)**	$f(x) = g(x), \forall x \in D$	