

SEM VI
MATHEMATICS PAPER IV
QUESTION BANK

UNIT I

- (1) The value of $\left(\frac{22}{7}\right)$ is
(a) π (b) 1 (c) -1 (d) 0
- (2) If $\left(\frac{2}{p}\right)=1$, then
(a) $p \equiv 1 \text{ or } 7 \pmod{8}$ (b) $p \equiv 3 \text{ or } 5 \pmod{8}$
(c) $p \equiv 1 \text{ or } 7 \pmod{12}$ (d) $p \equiv 3 \text{ or } 5 \pmod{12}$
- (3) If $\left(\frac{3}{p}\right)=1$, then
(a) $p \equiv 1 \text{ or } 3 \pmod{12}$ (b) $p \equiv 1 \text{ or } 7 \pmod{12}$
(c) $p \equiv 1 \text{ or } 5 \pmod{12}$ (d) $p \equiv 1 \text{ or } 11 \pmod{12}$
- (4) If $\left(\frac{-3}{p}\right)=1$, then
(a) $p=6k+1$ (b) $p=6k+5$ (c) $p=8k+5$ (d) cannot say
- (5) If $p=97$, then
(a) $\left(\frac{-1}{p}\right)=1 \text{ and } \left(\frac{2}{p}\right)=1$ (b) $\left(\frac{-1}{p}\right)=-1 \text{ and } \left(\frac{2}{p}\right)=-1$
(c) $\left(\frac{-1}{p}\right)=-1 \text{ and } \left(\frac{2}{p}\right)=1$ (d) $\left(\frac{-1}{p}\right)=1 \text{ and } \left(\frac{2}{p}\right)=-1$
- (6) Which of the following is correct:
(a) $\left(\frac{10}{83}\right)=1 \text{ and } \left(\frac{10}{53}\right)=1$ (b) $\left(\frac{10}{83}\right)=-1 \text{ and } \left(\frac{10}{53}\right)=-1$
(c) $\left(\frac{10}{83}\right)=1 \text{ and } \left(\frac{10}{53}\right)=-1$ (d) $\left(\frac{10}{83}\right)=-1 \text{ and } \left(\frac{10}{53}\right)=1$
- (7) Which of the following is correct:
(a) $\left(\frac{15}{97}\right)=1 \text{ and } \left(\frac{15}{61}\right)=1$ (b) $\left(\frac{15}{97}\right)=-1 \text{ and } \left(\frac{15}{61}\right)=-1$

(c) $\left(\frac{15}{97}\right) = 1$ and $\left(\frac{15}{97}\right) = -1$

(d) $\left(\frac{15}{97}\right) = -1$ and $\left(\frac{15}{61}\right) = 1$

(8) if $1 < a < p$, then the value of $\left(\frac{a^{p-1}}{p}\right)$ is

(a) 1

(b) a

(c) $p-1$

(d) p

(9) The value of $\left(\frac{2^{40}}{41}\right)$ is

(a) 4 1

(b) 40

(c) 2

(d) 1

(10) The value of $\left(\frac{2^5}{41}\right)$ is

(a) 5

(b) 2

(c) 1

(d) -1

(11) The value of $\left(\frac{(p-1)!}{p}\right)$ is

(a) $(-1)^{\frac{p-1}{2}}$

(b) $(-1)^{\frac{p+1}{2}}$

(c) $(p-1)!$

(d) None of these

(12) The value of $\sum_{j=1}^{p-1} \left(\frac{j}{p}\right)$ is

(a) 1

(b) -1

(c) 0

(d) $p-1$

(13) In which of the following case, both congruence equations have solutions:

(a) $x^2 \equiv 3 \pmod{5}, x^2 \equiv 5 \pmod{3}$

(b) $x^2 \equiv 3 \pmod{7}, x^2 \equiv 7 \pmod{3}$

(c) $x^2 \equiv 5 \pmod{11}, x^2 \equiv 11 \pmod{5}$

(d) $x^2 \equiv 5 \pmod{13}, x^2 \equiv$

$13 \pmod{5}$

(14) If g is primitive root of odd prime p then which of the following is true:

(a) g is quadratic residue of p

(b) g is quadratic non residue of p

(c) g^{p-1} is quadratic non residue of p

(d) g^{p-2} is quadratic residue of p

(15) The prime p for which $\left(\frac{10}{p}\right)=1$ is

(a) $p \equiv 19 \pmod{40}$

(b) $p \equiv 7 \pmod{40}$

(c) $p \equiv 1 \pmod{40}$

(d) $p \equiv 33 \pmod{40}$

16) If $p=7$ and $q=13$, then

(a) $\left(\frac{-1}{pq}\right) = 1$ and $\left(\frac{2}{pq}\right) = -1$

(b) $\left(\frac{-1}{pq}\right) = -1$ and $\left(\frac{2}{pq}\right) = 1$

(c) $\left(\frac{-1}{pq}\right) = 1$ and $\left(\frac{2}{pq}\right) = 1$

(d) $\left(\frac{-1}{pq}\right) = -1$ and $\left(\frac{2}{pq}\right) = -1$

(17) Let a, b be positive integers which are relatively prime and $b > 1$ be odd, then

(a) a is quadratic residue of b if and only if $\left(\frac{a}{b}\right)=1$.

(b) If a is quadratic residue of b then $\left(\frac{a}{b}\right)=1$.

(c) If $\left(\frac{a}{b}\right)=1$, then a is quadratic residue of b .

(d) None of these

(18) The congruence $x^2 \equiv a \pmod{32}$ (with $1 \leq a \leq 31$) is solvable for

(a) $a = 1, 9, 17, 25$ only

(b) $a = 1, 5, 9, 25$ only

(c) $a = 1, 5, 9, 21, 25$ only

(d) $a = 1, 21, 25$ only

(19) Let p be an odd prime. The congruence $x^2 + \left(\frac{p+1}{4}\right) \equiv 0 \pmod{p}$

(a) Is solvable if p is of the type $4k+3$

(b) Is not solvable if p is of the type $4k+3$

(c) Is solvable if p is of the type $8k+7$

(d) None of these

(20) Let p be a prime. There exist integers x, y with $(x, p)=1, (y, p)=1$ and $x^2 + y^2 \equiv 0 \pmod{p}$

(a) For all prime p

(b) For all primes of the type $4k+3$

(c) Only for $p = 2$

(d) For $p=2$ and primes of the type $4k+3$

(21) The number of solutions of the congruence $x^2 \equiv 3 \pmod{11^2 23^2}$ is

(a) 0

(b) 2

(c) 4

(d) 1

(22) The congruence $x^2 \equiv 231 \pmod{1105}$ has

(a) 2 solutions

(b) 1 solution

(c) 4 solutions

(d) no solutions

(23) The congruence $x^2 \equiv 25 \pmod{1013}$ has

(a) 2 solutions

(b) 1 solution

(c) 4 solutions

(d) no solutions

(24) Which of the following is correct?

(a) The quadratic congruence $x^2 \equiv 12 \pmod{5}$ has a solution.

(b) The quadratic congruence $x^2 \equiv 12 \pmod{7}$ has a solution.

(c) The quadratic congruence $x^2 \equiv 12 \pmod{35}$ has a solution.

(d) None of these.

(25) Which of the following is false?

(a) $x^2 \equiv a \pmod{2}$ always has a solution.

- (b) $x^2 \equiv a \pmod{4}$ has solution if and only if $a \equiv 1 \pmod{4}$
- (c) $x^2 \equiv a \pmod{2^n}$, for $n > 2$ has a solution if and only if $a \equiv 1 \pmod{8}$
- (d) None of (a),(b),(c) is true .
- (26) If $x^2 \equiv a \pmod{2^n}$, for $n > 2$ has a solution then it has
- (a) exactly 2 incongruent solutions (b) exactly 4 incongruent solutions
- (c) exactly 1 solution (d) none of these
- (27) The congruence $x^2 \equiv 19 \pmod{7^3}$ has
- (a) only one solution (b) two solutions (c) no solution (d) none of these

UNIT II

- 1) The initial integer in the symbol $[a_0, a_1, \dots, a_n]$ will be zero when the value of the fraction is
- (a) *Positive & Greater than one* (b) *Positive & less than one*
- (c) *Negative* (d) Can not say
- 2) The simple continued fraction (SCF) for $\frac{135}{79}$ is given by
- (b) $[0, 1, 1, 2, 2, 3, 3]$ (b) $[2, 1, 2, 1, 3, 3]$ (c) $[1, 1, 2, 2, 3, 3]$ (d) None of these
- (3) The SCF for $-\frac{73}{116}$ is given by
- (a) $[-1, 2, 1, 2, 3, 4]$ (b) $[-2, 1, 1, 2, 3, 4]$ (c) $[-1, 1, 2, 1, 2, 3]$ (d) None of these
- (4) The SCF $[0, 1, 2, 3, 4, 3]$ represents
- (a) $\frac{97}{135}$ (b) $\frac{97}{139}$ (c) $\frac{34}{139}$ (d) None of these
- (5) The SCF $[-2, 1, 2, 3, 4, 3]$ represents
- (a) $-\frac{181}{139}$ (b) $-\frac{97}{139}$ (c) $-\frac{34}{139}$ (d) None of these
- (6) The SCF $[2, 1, 2, 1, 2, 1, 2]$ equals
- (a) $[1, 1, 1, 2, 1, 2, 1, 2]$ (b) $[2, 1, 2, 1, 2, 1, 2, 1]$ (c) $[2, 1, 2, 1, 2, 1, 1, 1]$ (d) None of these
- (7) The SCF $[2, 1, 2, 1, 2, 2, 1]$ equals
- (a) $[2, 1, 2, 1, 2, 1, 1, 1]$ (b) $[2, 1, 2, 1, 2, 3]$ (c) $[1, 1, 2, 1, 2, 2, 1]$ (d) None of these

(8) If $r = [2, 3, 3, 2]$ then $\frac{1}{r}$ is given by

- (a) $[2, 3, 3, 2]$ (b) $[\frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{2}]$ (c) $[0, 2, 3, 3, 2]$ (d) None of these

(9) The value of the 4th convergent of $[2, 3, 1, 4, 2, 3]$ is

- (a) $\frac{95}{42}$ (b) $\frac{43}{19}$ (c) 2 (d) None of these

(10) If u_6/u_5 represents quotient of two successive numbers in the Fibonacci Sequence then $u_6/u_5 =$

- (a) $[2, 2, 2, 2, 2]$ (b) $[1, 1, 1, 1, 1]$ (c) $[-1, 1, 1, 1, 1]$ (d) None of these

(11) Which of the following statement is correct :

- (a) $p_0 = a_0; q_0 = 1$ (b) $p_0 = 1; q_0 = 1$ (c) $p_0 = 1; q_0 = a_0$ (d) $p_0 = a_1; q_0 = a_0$

(12) Which of the following statement is correct :

- (a) $p_1 = a_1; q_1 = 1$ (b) $p_1 = 1; q_1 = a_1$ (c) $p_1 = a_1 a_0 + 1; q_1 = a_1$ (d) $p_1 = 1; q_1 = a_0$

(13) For $k \geq 1$ which of the following statement is correct :

- (a) $p_k q_{k-1} - q_k p_{k-1} = (-1)^k$ (b) $p_k q_{k-1} - q_k p_{k-1} = (-1)^{k-1}$
(c) $p_k q_{k-1} - q_k p_{k-1} = (-1)^k a_k$ (d) $p_k q_{k-1} - q_k p_{k-1} = (-1)^{k-1} a_k$

(14) For $k \geq 2$ which of the following statement is correct :

- (a) $p_k q_{k-2} - q_k p_{k-2} = (-1)^{k-2}$ (b) $p_k q_{k-2} - q_k p_{k-2} = (-1)^k$
(c) $p_k q_{k-2} - q_k p_{k-2} = (-1)^{k-2} a_k$ (d) $p_k q_{k-2} - q_k p_{k-2} = (-1)^k a_k$

(15) Which of the following statement is correct :

- (a) $C_0 > C_2 > C_4 > C_6 \dots$ (b) $C_1 < C_3 < C_5 < C_7 \dots$ (c) $C_0 < C_2 < C_4 < C_6 \dots$ (d) $C_1 \leq C_3 \leq C_5 \leq C_7 \dots$

(16) For a positive integer 'c' ; if $SCF [a_0, a_1, \dots, a_n] > [a_0, a_1, \dots, a_n + c]$ then

- (a) n is odd (b) n is even (c) Both (a) & (b) (d) None of (a) & (b)

17) The SICF of $\sqrt{15}$ is given by

- (a) $[3, \overline{1, 3}]$ (b) $[3, \overline{1, 6}]$ (c) $[3, 1, 2, 3, 4, 8, \overline{5}]$ (d) None of these

18) The SICF of $\sqrt{2} - 1$ is given by

(a) $[0, \overline{1,2}]$ (b) $[\overline{1,3}]$. (c) $[0, \overline{2}]$ (d) None of these

(19) If $\alpha = [\overline{2,1}]$ then α equals

(a) $1 + \sqrt{3}$ (b) $2^{1/2}$ (c) $1 - \sqrt{3}$ (d) None of these

(20) The SICF of $\frac{(e-1)}{(e+1)}$ is given by

(a) $[e, 1, e, -1]$ (b) $[0, 2, 6, 10, 14, 18, \dots]$ (c) $[2, 1, 2, 1, 4, 1, 8, \dots]$ (d) None of these

(21) The SICF of $\frac{(e^2-1)}{(e^2+1)}$ is given by

(a) $[0, 1, 3, 5, 7, 9, \dots]$ (b) $[0, 2, 6, 10, 14, 18, \dots]$
(c) $[1, 1, 4, 5, 7, 8, \dots]$ (d) None of these

(22) The SICF $[1, 1, 1, 1, \dots]$ represents

(a) 1 (b) 1.1111 (c) $\frac{1+\sqrt{5}}{2}$ (d) None of these

(23) For $n \in \mathbb{N}$, $\sqrt{n^2 + 1} =$

(a) $[n, \overline{n, 2n}]$ (b) $[n, \overline{2n}]$ (c) $[n, \overline{1, 2n}]$ (d) None of these

(24) Let $x = [1, 3, 1, 5, 1, 7, 1, 9, \dots]$. If $C_n = \frac{p_n}{q_n}$ is the n^{th} convergent of x then we know that $|x - C_n| < \frac{1}{q_n q_{n+1}}$, using this inequality the rational approximation to x correct upto 3 decimal places is

(a) $\frac{34}{27}$ (b) $\frac{301}{239}$ (c) $\frac{267}{212}$ (d) None of these

(25) If $x = \frac{1+\sqrt{13}}{2}$ then $x =$

(a) $[2, \overline{3}]$ (b) $[2, \overline{1, 3}]$ (c) $[2, 1, \overline{3}]$. (d) None of these.

(26) If $\alpha = [a_0, a_1, a_2, \dots]$ and $C_n = [a_0, a_1, a_2, \dots, a_n]$ is the n^{th} convergent then $\alpha =$

(a) $\lim_{n \rightarrow \infty} C_n$ (b) $\lim_{n \rightarrow \infty} C_{n-1}$ (c) Both (a) and (b) (d) None of these.

(27) If $C_k = \frac{p_k}{q_k}$ is the k^{th} convergent of SICF $[a_0, a_1, a_2, \dots]$ then

(a) $(C_{10}, C_{11}) \subseteq (C_2, C_3)$ (b) $(C_{10}, C_{11}) \subseteq (C_{12}, C_3)$

- (c) $(C_{10}, C_{11}) \subseteq (C_8, C_3)$ (d) none of these

UNIT III

1) Which of the following is the solution of $\tau(n) = 4$?

- (a) 2 (b) 4 (c) 8 (d) 16

2) Which of the following is the solution of $\sigma(n) = 4$?

- (a) 1 (b) 2 (c) 3 (d) 4

(3) $\sigma(n) = 2^k, k \in \mathbb{N}$ has no solution for

- (a) $k=1$ (b) $k=2$ (c) $k=3$ (d) $k=5$

(4) If the prime $p \equiv -1 \pmod{4}$ and if $2|k$, then $\sigma(p^k)$ is congruent 4 to

- (a) $0 \pmod{4}$ (b) $1 \pmod{4}$ (c) $2 \pmod{4}$ (d) cannot say

(5) Which of the following is not perfect?

- (a) $2^2(2^3 - 1)$ (b) $2^4(2^5 - 1)$ (c) $2^6(2^7 - 1)$ (d) $2^{10}(2^{11} - 1)$

(6) Let p and q be distinct primes and $n = pq$. Then $\sigma(n)$ is

- (a) pq (b) $(p+1)(q+1)$ (c) $n+1$ (d) None

(7) Let p and q be distinct primes and $n = pq$. Then $\tau(n)$ is

- (a) 1 (b) 2 (c) 4 (d) $p+q$

(8) If $2^n - 1$ and $2^n + 1$ are both primes for $n \in \mathbb{N}$, then

- (a) n must be odd (b) there are infinitely many such n
(c) $n=2$ (d) None of the above

(9) Let $F_n = 2^{2^n} + 1$, then for $n \neq m$, $\gcd(F_n, F_m)$ is

- (a) n (b) 2^m (c) 1 (d) None

(10) Let p and q be distinct primes and $M_n = 2^n - 1$. Then $\gcd(M_p, M_q)$ is

- (a) 1 (b) p (c) q (d) n

11) The fundamental solution of $x^2 - 3y^2 = 1$ is

- (a) (2, 1) (b) (7, 4) (c) (1, 0) (d) None

12) Pell's equation $x^2 - 13y^2 = -1$ has

- (d) Only one solution (b) no solution
(c) infinitely many solutions (d) None

(13) Let $\sqrt{29} = [a_0; \overline{a_1, a_2, \dots, a_n}]$. Then $(a_n - a_{n-1} + a_{n-2} - \dots + a_2 - a_1)$ equals

- (a) 29 (b) -1 (c) 3 (d) None

(14) Pell's equation $x^2 - 30y^2 = -1$ has

- (a) only one solution (b) no solution (c) infinitely many solutions (d) None

(15) If $\alpha = 1 + \sqrt{2}$, $\beta = 1 - \sqrt{2}$ then the Pell numbers P_n and Q_n are given as

- (a) $P_n = \frac{(\alpha^n + \beta^n)}{2\sqrt{2}}$, $Q_n = \frac{(\alpha^n - \beta^n)}{2}$ (b) $Q_n = \frac{(\alpha^n + \beta^n)}{2\sqrt{2}}$, $P_n = \frac{(\alpha^n - \beta^n)}{2}$
(c) $P_n = \frac{(\alpha^n - \beta^n)}{2\sqrt{2}}$, $Q_n = \frac{(\alpha^n + \beta^n)}{2}$ (d) None

(16) For the Pell numbers P_n and Q_n with $n \geq 1$, $Q_n - P_n =$

- (a) P_{n-1} (b) Q_{n-1} (c) $2P_{n-1}$ (d) None

(17) If x_1, y_1 is a fundamental solution of $x^2 - dy^2 = 1$, then every positive solution of the equation is given by x_n, y_n which satisfy

- (a) $y_n + x_n \sqrt{d} = (x_1 + y_1 \sqrt{d})^n$ (b) $x_n + y_n \sqrt{d} = (x_1 + y_1 \sqrt{d})^n$
(c) Both (a) and (b) (d) None

(18) if $C_n = \frac{p_n}{q_n}$ ($n=0,1,2,\dots$) is the n^{th} convergent and k is the length of the period

Of the infinite SCF of \sqrt{d} , then p_{nk-1}, q_{nk-1} is a solution of $x^2 - dy^2 = 1$

- (a) $k=2, n=5$ (b) $k=3, n=4$ (c) Both (a) and (b) (d) None

(19) Every Carmichael number, an absolute pseudoprime

- (a) is odd (b) has atleast 3 distinct prime factors (c) composite
(d) (a), (b), and (c)

(20) If d is divisible by a prime $p \equiv 3 \pmod{4}$, then the equation $x^2 - dy^2 = -1$ has

- (a) no solution (b) infinitely many solutions (c) one solution (d) None

