TYBSC, MATHS PAPER III, METRIC SPACES, QUESTION BANKS
(SEQUENCES AND SERIES)

	Choose correct alternative in each of the following				
1	**Let** $f_n(x) = sin\ sin\ nx$ **for** $x \in R$ **and** $g_n(x) = \frac{f_n(x)}{n}$ $\forall\ x \in R$**. Then**				
	(a)	$\{f_n\}$ **and** $\{g_n\}$ **are uniformly convergent on** R			
	(b)	$\{f_n\}$ **and** $\{g_n\}$ **are not pointwise convergent on** R			
	(c)	$\{g_n\}$ **is uniformly convergent on** R **but** $\{f_n\}$ **is not**			
	(d)	$\{f_n\}$ **is uniformly convergent on** R **but** $\{g_n\}$ **is not**			
2	**If** $\{f_n\}$ **is a sequence of real valued continuous functions defined on** $[a,b]$ **and converging uniformly to** f **on** $[a,b]$ **then**				
	(a)	f **may not be continuous on** $[a,b]$	**(b)**	f **is not bounded on** $[a,b]$	
	(c)	f **is integrable on** $[a,b]$	**(d)**	f **is not integrable on** $[a,b]$	
3	**Let** $f_n: [0,1] \to [0,1]$ **be defined by** $f_n(x) = x * \chi_n(x)$ **where** $\chi_n(x) = \{0\ if\ x \notin \left[0,\frac{1}{n}\right]\ 1\ if\ x \in \left[0,\frac{1}{n}\right]$				
	(a)	$\{f_n\}$ **converges uniformly to** 0 **on** $[0,1]$			
	(b)	$\{f_n\}$ **converges pointwise to** 1 **on** $[0,1]$ **but does not converge uniformly**			
	(c)	$\{f_n\}$ **converges uniformly to** 1 **on** $[0,1]$			
	(d)	$\{f_n\}$ **converges uniformly to** x **on** $[0,1]$			
4	**Let** $\{f_n\}$ **and** $\{g_n\}$ **be sequences of real valued bounded functions defined on** $[a,b]$**. If** $\{f_n\}$ **and** $\{g_n\}$ **converge uniformly to** f **and** g **respectively then**				
	(a)	$\{f_n * g_n\}$ **converges to an unbounded function** $f * g$			
	(b)	$\{f_n * g_n\}$ **converges to a bounded function** $f * g$			
	(c)	$\{f_n * g_n\}$ **does not converge uniformly to** $f * g$			
	(d)	$\{f_n * g_n\}$ **may not be pointwise convergent on** $[a,b]$			
5	**Let** $f_n(x) = x^{n-1}(1-x)$, $0 \le x \le 1$ **then**				
	(a)	$\{f_n\}$ **is uniformly convergent on** $[0,1]$			
	(b)	$\{f_n\}$ **is not uniformly convergent on** $[0,1]$			
	(c)	$\{f_n\}$ **is not pointwise convergent on** $[0,1]$			
	(d)	$\{f_n\}$ **converges pointwise to an unbounded function**			
6	**Let** $f_n(x) = \frac{x}{1+nx^2}$ **,** $x \in R$				
	(a)	$\{f_n\}$ **converges uniformly on** R **but** $\{f_n'\}$ **does not converge uniformly on** R			

	(b)	$\{f_n\}$ **converges uniformly on** R **and** $\{f_n'\}$ **also converges uniformly on** R		
	(c)	$\{f_n\}$ **does not converge uniformly on** R **but** $\{f_n'\}$ **converges uniformly on** R		
	(d)	**Neither** $\{f_n\}$ **nor** $\{f_n'\}$ **converge uniformly on** R		
7	**Let** $\{f_n\}$ **be a sequence of real valued functions defined on** $[a,b]$ **converging uniformly to a function** f **on** $[a,b]$ **then the following statement is not true**			
	(a)	**Each** f_n **is bounded on** $[a,b] \Rightarrow f$ **is bounded on** $[a,b]$		
	(b)	**Each** f_n **is differentiable on** $(a,b) \Rightarrow f$ **is differentiable on** (a,b)		
	(c)	**Each** f_n **is continuous on** $[a,b] \Rightarrow f$ **is continuous on** $[a,b]$		
	(d)	**Each** f_n **is integrable on** $[a,b] \Rightarrow f$ **is integrable on** $[a,b]$		
8	**Let** $f_n(x) = \{nx \; if \; 0 \le x \le \frac{1}{n} \; 1 \;\; if \; \frac{1}{n} < x \le 1$ **then**			
	(a)	$\{f_n\}$ **converges pointwise to** 0	**(b)**	$\{f_n\}$ **converges uniformly to** 0
	(c)	$\{f_n\}$ **converges uniformly to** 1	**(d)**	$\{f_n\}$ **is not uniformly convergent on** $[0,1]$
9	**Let** $\{f_n\}$ **be a sequence of real valued differentiable functions defined on** (a,b)**. Let** $f_n(x) = f(x)$ **and** $f_n'(x) = g(x)$ **(pointwise limits)**			
	(a)	**If** f **is differentiable on** (a,b) **then** $f' = g$ **on** (a,b)		
	(b)	**If** $\{f_n'\}$ **converges uniformly to** g **then** f **is differentiable on** (a,b) **and** $f' = g$		
	(c)	**If** $f' = g$ **on** (a,b) **then** $\{f_n\}$ **converges uniformly to** f **on** (a,b)		
	(d)	**If** $\{f_n\}$ **converges uniformly to** f **then** f **is differentiable and** $f' = g$ **on** (a,b)		
10	**Let** $\{f_n\}$ **be a sequence of real valued Riemann integrable functions defined on** $[a,b]$ **and** f **be the pointwise limit of** $\{f_n\}$			
	(a)	**If** $\int_a^b f_n \neq \int_a^b f$ **then** $\{f_n\}$ **does not converge uniformly to** f		
	(b)	**If** $\{f_n\}$ **does not converge uniformly to** f **then** $\int_a^b f_n \neq \int_a^b f$		
	(c)	**If** $\int_a^b f_n \neq \int_a^b f$ **then the convergence is uniform**		
	(d)	**If** $\int_a^b f_n = \int_a^b f$ **then** $\{f_n\}$ **converges uniformly to** f		
11	**If** $\{f_n\}$ **and** $\{g_n\}$ **are sequences of real valued functions defined on a nonempty subset** S **of** R **which converge uniformly to** f **and** g **respectively then**			
	(a)	$\{f_n * g_n\}$ **is uniformly convergent to** $f * g$ **on** S **if each** f_n **is bounded on** S		
	(b)	$\{f_n + g_n\}$ **is not uniformly convergent on** S		
	(c)	$\{f_n * g_n\}$ **need not be uniformly convergent on** S		
	(d)	$\{f_n * g_n\}$ **converges uniformly to** $f * g$ **on** S **iff either** $f \equiv 0$ **or** $g \equiv 0$ **on** S		
12	**Let** $f_n(x) = \frac{nx}{1+n^2x^2}$ **,** $\forall\, n \in N$ **and** $x \in R$ **then** $\{f_n\}$ **is not uniformly convergent on**			
	(a)	$[1,2]$	**(b)**	$[-2,-1]$

	(c)	$(1,2)$	**(d)**	$[0,1]$
13	**If $\{f_n\}$ is a sequence of real valued uniformly continuous functions defined on R converging uniformly to f on R then**			
	(a)	**f may not be uniformly continuous on R**		
	(b)	**f is uniformly continuous on R**		
	(c)	**f may not be continuous on R**		
	(d)	**f is not bounded on R**		
14	**Let $f_n(x) = \frac{x^n}{1+x^n}$, $\forall\, n \in N$ and $\forall\, x \in [0,2]$. If $f(x) = f_n(x)$ then**			
	(a)	**$\{f_n\}$ converges uniformly to f on $[0,2]$ and f is continuous at $x = 1$**		
	(b)	**$\{f_n\}$ does not converge uniformly to f on $[0,2]$ but f is continuous on $[0,1]$**		
	(c)	**$\{f_n\}$ does not converge uniformly to f on $[0,2]$ and f is not continuous on $[0,1]$**		
	(d)	**$\{f_n\}$ converges uniformly to f on $[0,2]$ and f is not continuous at $x = 1$**		
15	**If $\{f_n\}$ is a sequence of real valued bounded functions defined on $[a, b]$ and converging uniformly to f on $[a, b]$ then**			
	(a)	**f is bounded on $[a, b]$**	**(b)**	**$glb\{f(x): x \in [a, b]\}$ does not exist in R**
	(c)	**f may not be bounded on $[a, b]$**	**(d)**	**$lub\{f(x): x \in [a, b]\}$ does not exist in R**
16	**Let $f_n(x) = \sqrt[n]{x}$, $\forall\, n \in N$ and $\forall\, x \in [0,1]$**			
	(a)	**$\{f_n\}$ converges uniformly on $[0,1]$**		
	(b)	**$\{f_n\}$ converges pointwise to a continuous function on $[0,1]$**		
	(c)	**$\{f_n\}$ converges pointwise to a discontinuous function on $[0,1]$**		
	(d)	**$\{f_n\}$ is not pointwise convergent on $[0,1]$**		
17	**Let $\{f_n\}$ be a sequence of real valued functions defined on $S \subseteq R$ and $A \subseteq S$. If $\{f_n\}$ does not converge uniformly on S then**			
	(a)	**$\{f_n\}$ does not converge uniformly on A**		
	(b)	**$\{f_n\}$ is not pointwise convergent on A**		
	(c)	**$\{f_n\}$ may converge uniformly on A**		
	(d)	**$\{f_n\}$ is not pointwise convergent on S**		
18	**A sequence $\{f_n\}$ of real valued functions defined on $S \subseteq R$ converges uniformly to a function $f: S \to R$. If $M_n = sup\ sup\ \{\lvert f_n(x) - f(x)\rvert : x \in S\}$ then**			
	(a)	**M_n is a finite number**	**(b)**	**M_n does not exist**
	(c)	$M_n = 0$	**(d)**	**(M_n) is an unbounded sequence**
19	**Let $f_n: R \to R$ be defined as $f_n(x) = \{0\ if\ x \notin [-n, n]\ 1\ if\ x \in [-n, n]$**			
	(a)	**$\{f_n\}$ converges uniformly to f where $f: R \to R$ is defined as $f(x) = 0$**		

	(b)	$\{f_n\}$ **converges uniformly to** f **where** $f: R \to R$ **is defined as** $f(x) = 1$
	(c)	$\{f_n\}$ **is not pointwise convergent on** R
	(d)	$\{f_n\}$ **does not converge uniformly on** R
20	**Let** $\{f_n\}$ **be a sequence of real valued functions defined on** $S \subseteq R$ **and** $\{f_n\}$ **converge to** f **pointwise on** S**. Suppose there is a sequence** (t_n) **of real numbers such that** $\lvert f_n(x) - f(x)\rvert \le t_n$ **for all** $n \in N$ **and for all** $x \in S$**. If** (t_n) **converges to** 0 **then**	
	(a)	$\{f_n\}$ **does not converge to** f **uniformly on** S
	(b)	**Can not say about the uniform convergence of** $\{f_n\}$
	(c)	$\{f_n\}$ **may converge uniformly to** f
	(d)	$\{f_n\}$ **converges to** f **uniformly on** S
21	**Let** $f(x) = \sum_{n=1}^{\infty} \frac{coscos\, nx}{n^2}$ **then**	
	(a)	$\sum_{n=1}^{\infty} \frac{coscos\, nx}{n^2}$ **is not uniformly convergent on** $[0,1]$ **and can not be integrated term by term**
	(b)	$\sum_{n=1}^{\infty} \frac{coscos\, nx}{n^2}$ **is uniformly convergent on** $[0,1]$ **and can be integrated term by term**
	(c)	$\sum_{n=1}^{\infty} \frac{coscos\, nx}{n^2}$ **is not uniformly convergent on** $[0,1]$ **but** $\int_0^1 \sum_{n=1}^{\infty} \frac{coscos\, nx}{n^2} = \sum_{n=1}^{\infty} \int_0^1 \frac{coscos\, nx}{n^2}$
	(d)	$\sum_{n=1}^{\infty} \frac{coscos\, nx}{n^2}$ **does not exist for some** $x \in R$
22	**If** $\sum_{n=1}^{\infty} f_n(x)$ **is a series of real valued continuous functions defined on** $[a,b]$ **and converging uniformly to** f **on** $[a,b]$ **then**	
	(a)	f **is not continuous on** $[a,b]$ **(b)** f **is not bounded on** $[a,b]$
	(c)	f **is not integrable on** $[a,b]$ **(d)** f **is bounded on** $[a,b]$
23	**The series** $\sum_{n=1}^{\infty} (-x)^n(1-x)$ **is**	
	(a)	**Uniformly convergent on** R
	(b)	**Uniformly convergent on** $[0,a]$ **where** $0 \le a < 1$ **but not on** $[0,1]$
	(c)	**Pointwise convergent on** R
	(d)	**Uniformly convergent on** $[0,1]$
24	**Let** $\sum_{n=1}^{\infty} f_n$ **be a series of real valued Riemann integrable functions defined on** $[a,b]$ **and** f **be the pointwise limit of** $\sum_{n=1}^{\infty} f_n$	
	(a)	**If** $\int_a^b f = \sum_{n=1}^{\infty} \int_a^b f_n$ **then** $\sum_{n=1}^{\infty} f_n$ **converges to** f **uniformly on** $[a,b]$

	(b)	**If** $\sum_{n=1}^{\infty} f_n$ **does not converge to** f **uniformly on** $[a,b]$ **then** $\int_a^b f \neq \sum_{n=1}^{\infty} \int_a^b f_n$
	(c)	$\int_a^b \sum_{n=1}^{\infty} f_n = \sum_{n=1}^{\infty} \int_a^b f_n$
	(d)	**If** $\sum_{n=1}^{\infty} f_n$ **converges to** f **uniformly on** $[a,b]$ **then** $\int_a^b f = \sum_{n=1}^{\infty} \int_a^b f_n$
25	**The series** $\sum_{n=1}^{\infty} \frac{x^2}{(1+x^2)^n}$	
	(a)	**Converges uniformly on** $(0,\infty)$
	(b)	**Converges uniformly on** $[a,\infty)$ **where** $a>0$
	(c)	**Does not converge uniformly on** $[a,\infty)$ **where** $a>0$
	(d)	**Converges uniformly on** $(0,a)$ **where** $a>0$
26	**If** $\{f_n\}$ **is a sequence of differentiable functions on** $[a,b]$ **such that each** $f_n{}'$ **is continuous on** $[a,b]$ **and** $\sum_{n=1}^{\infty} f_n$ **converges to** f **pointwise on** $[a,b]$ **then**	
	(a)	$\sum_{n=1}^{\infty} f_n$ **converges to** f **uniformly on** $[a,b]$ **implies** $\frac{d}{dx}\sum_{n=1}^{\infty} f_n(x) = \sum_{n=1}^{\infty} \frac{d}{dx} f_n(x)$
	(b)	$\frac{d}{dx}\sum_{n=1}^{\infty} f_n(x) = \sum_{n=1}^{\infty} \frac{d}{dx} f_n(x)$ **implies** $\sum_{n=1}^{\infty} f_n$ **converges to** f **uniformly on** $[a,b]$
	(c)	$\frac{d}{dx}\sum_{n=1}^{\infty} f_n(x) = \sum_{n=1}^{\infty} \frac{d}{dx} f_n(x)$
	(d)	$\sum_{n=1}^{\infty} f_n{}'$ **converges uniformly on** $[a,b]$ **implies** $\frac{d}{dx}\sum_{n=1}^{\infty} f_n(x) = \sum_{n=1}^{\infty} \frac{d}{dx} f_n(x)$
27	**The series** $\sum_{n=1}^{\infty} x^n(1-x)$	
	(a)	**Converges uniformly on** $[0,1]$
	(b)	**Is not pointwise convergent on** $[0,1]$
	(c)	**Is uniformly convergent on** $[0,a]$ **where** $0<a<1$
	(d)	**Is uniformly convergent on** $[a,1]$ **where** $0<a<1$
28	**The series** $\sum_{n=1}^{\infty} \frac{x^n}{x^n+1}$	
	(a)	**Pointwise convergent on** $[1,\infty)$
	(b)	**Uniformly convergent on** $[0,\infty)$
	(c)	**Uniformly convergent on** $[0,a]$ **where** $a<1$
	(d)	**Uniformly convergent on** $[a,\infty)$ **where** $0<a<1$
29	**The series** $\sum_{n=1}^{\infty} \frac{nx^2}{n^3+x^3}$ **is**	
	(a)	**Uniformly convergent on** $[0,a]$ **where** $a>0$ **but not on** $[0,\infty)$
	(b)	**Not uniformly convergent on** $[0,a]$ **where** $a>0$
	(c)	**Uniformly convergent on** $[0,\infty)$
	(d)	**Uniformly convergent on** $[a,\infty)$ **where** $0<a<1$

30	If $\sum_{n=1}^{\infty} \quad \|a_n\|$ is convergent then $\sum_{n=1}^{\infty} \quad a_n x^n$ is			
	(a)	Uniformly convergent on R		
	(b)	Uniformly convergent on any closed and bounded interval		
	(c)	Uniformly convergent on $[-a, a]$ where $0 \leq a < 1$		
	(d)	Pointwise convergent on any closed and bounded interval		
31	If the power series $\sum_{n=0}^{\infty} \quad c_n x^n$ converges at $x = 1$ and diverges at $x = 2$ then the power series $\sum_{n=0}^{\infty} \quad \|a_n x^n\|$ converges			
	(a)	For all $x \in R$ with $\|x\| < 2$ and diverges for all $x \in R$ with $\|x\| > 2$		
	(b)	For all $x \in R$ with $\|x\| < 1$ and diverges for all $x \in R$ with $1 < \|x\| < 2$		
Ans	(c)	For all $x \in R$ with $\|x\| < 1$ and diverges for all $x \in R$ with $\|x\| > 2$		
	(d)	For all $x \in R$ with $1 < \|x\| < 2$		
32	If the power series $\sum_{n=0}^{\infty} \quad c_n x^n$ has radius of convergence 1 then			
	(a)	The power series converges at $x = 1$ and $x = -1$		
	(b)	The power converges at $x = 1$ and diverges at $x = -1$		
	(c)	The power diverges at $x = 1$ and converges at $x = -1$		
	(d)	Can not say about the convergence and divergence at $x = 1$ and $x = -1$		
33	If R is the radius of convergence of the power series $\sum_{n=0}^{\infty} \quad c_n x^n$ then the radius of convergence of the series $\sum_{n=1}^{\infty} \quad n c_n x^{n-1}$ is			
	(a)	R	(b)	$\sqrt{R}$
	(c)	$R + 1$	(d)	R^2
34	$\sum_{n=0}^{\infty} \quad a_n x^n$ has radius of convergence R_1 and $\sum_{n=0}^{\infty} \quad b_n x^n$ has radius of convergence R_2 Let $c_n = \{a_n \; if \; n \; is \; even \; b_n \; if \; n \; is \; odd$ then the radius of convergence of the power series $\sum_{n=0}^{\infty} \quad c_n x^n$ is			
	(a)	$R_1 + R_2$	(b)	$\{R_1, R_2\}$
	(c)	$\{R_1, R_2\}$	(d)	$R_1 - R_2$
35	Let $f(x) = \sum_{n=0}^{\infty} \quad c_n x^n$ for $\|x\| < R$. If $f(x)$ is an even function then			
	(a)	$c_n = 0$ for all $n \in N$	(b)	$c_n = 0$ when n is odd
	(c)	$c_n = 0$ when n is even	(d)	$c_n \neq 0$ for any $n \in N$
36	If R is the radius of convergence of the power series $\sum_{n=0}^{\infty} \quad c_n x^n$ then radius of convergence of the power series $\sum_{n=0}^{\infty} \quad c_n x^{nk}$ is			
	(a)	R^k	(b)	R
	(c)	$R^{1/k}$	(d)	$\frac{1}{R^k}$
37	Consider the power series $\sum_{n=1}^{\infty} \quad \frac{(-1)^n x^n}{n}$ then			
	(a)	Radius of convergence $= 1$ and interval of convergence is $[-1,1]$		
	(b)	Radius of convergence $= 1$ and interval of convergence is $(-1,1)$		

	(c)	**Radius of convergence** $= 1$ **and interval of convergence is** $[-1,1)$		
T	**(d)**	**Radius of convergence** $= 1$ **and interval of convergence is** $(-1,1]$		
38	**If** α **is a non-zero real number then the radius of convergence of the power series** $\sum_{n=0}^{\infty} \alpha^n x^n$ **is**			
	(a)	$\lvert\alpha\rvert$	**(b)**	$\lvert\alpha\rvert^{1/2}$
	(c)	$\frac{1}{\lvert\alpha\rvert}$	**(d)**	∞
39	**The series expansion** $log\, log\,(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots\cdots$ **is valid if**			
	(a)	$\lvert x\rvert \leq 1$	**(b)**	$\lvert x\rvert \leq A$ **for** $A > 0$
	(c)	$\lvert x\rvert < 1$	**(d)**	$x > 0$
40	**The series expansion** $1 + 2x + 3x^2 + \cdots\cdots + nx^{n-1} + \cdots = \frac{1}{(1-x)^2}$ **is valid in**			
	(a)	R	**(b)**	$(-1,1)$
	(c)	$[-1,1)$	**(d)**	**Any closed and bounded interval**