Normal subgroups and Quotient groups, Cayley's theorem and external direct product of groups

1. Let $H_1 = \{I, (12)\}$ and $H_2 = \{I, (123), (132)\}$. Then
   (a) H_1, H_2 are normal subgroups of S_3.
   (b) H_1 is a normal subgroup of S_3 but H_2 is not a normal subgroup of S_3.
   (c) H_1, H_2 are not normal subgroups of S_3
   (d) H_2 is a normal subgroup of S_3 but H_1 is not a normal subgroup of S_3.

2. Let $H_1 = \{\sigma \in S_n : \sigma(n) = n\}, H_2 = \{\sigma \in S_n : \sigma(k) = k, \text{ for some } k,\ 1 \leq k \leq n\}$. Then
   (a) H_1, H_2 are normal subgroups of S_n.
   (b) H_1 is a normal subgroup of S_n but H_2 is not a normal subgroup of S_n.
   (c) H_1, H_2 are not normal subgroups of S_n
   (d) H_2 is a normal subgroup of S_n but H_1 is not a normal subgroup of S_n.

3. Let $G = \dfrac{\mathbb{Z}}{20\mathbb{Z}}, H = \dfrac{4\mathbb{Z}}{20\mathbb{Z}}$ (under addition). Then order of quotient group $\dfrac{G}{H}$ is

   (a) 4 (b) ∞ (c) 5 (d) 20

4. Let H be a normal subgroup of G. Let $|aH| = 3$ in $\frac{G}{H}$ and $\circ(H) = 10$, then order of a is
   (a) 1 (b) 30
   (c) one of 3, 6, 15 or 30 (d) none of these.

5. Let G be a group of order 5. If $\Phi : \mathbb{Z}_{30} \to G$ is a group homomorphism, then ker Φ has order
   (a) 5 (b) 30 or 6 (c) 30 or 5 (d) 1

6. Let G be a finite group. If $f_1 : G \to \mathbb{Z}_{10}$ and $f_2 : G \to \mathbb{Z}_{15}$ are onto group homomorphisms, then order of G is
   (a) $30k$, where $k \in \mathbb{N}$ (b) 5^k, where $k \in \mathbb{N}$ (c) 10 or 15 (d) 5

7. In the quotient group $\dfrac{\mathbb{Z}_{18}}{<\bar{6}>}$ (under addition), the order of the element $\bar{5}+<\bar{6}>$ is

   (a) 5 (b) 6 (c) 2 (d) 3

8. Let $G = GL_2(\mathbb{R}), K = \left\{\begin{pmatrix} a & b \\ 0 & d \end{pmatrix} : a, b, d \in \mathbb{R}, ad \neq 0\right\}, H = \left\{\begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} : b \in \mathbb{R}\right\}$. Then
   (a) H is a normal subgroup of K and K is a normal subgroup of G.
   (b) H is a normal subgroup of K but K is not a normal subgroup of G.
   (c) H is a not normal subgroup of K but K is a normal subgroup of G.
   (d) None of these.

9. Let H be a normal subgroup of a finite group G. If $|H| = 2$ and G has an element of order 3 then
   (a) G has a cyclic subgroup of order 6.

(b) G has a non-abelian subgroup of order 6.

(c) G has subgroup of order 4.

(d) None of these.

10. Let G be a group of order 30. If $Z(G)$ has order 5, then
(a) $\dfrac{G}{Z(G)}$ is cyclic. (b) $\dfrac{G}{Z(G)}$ is abelian but not cyclic.
(c) $\dfrac{G}{Z(G)}$ is non-abelian. (d) None of these.

11. Let $G = GL_2(\mathbb{R})$, $H = \{A \in G : detA \in \mathbb{Q}\}$, then
(a) H is a normal subgroup of G. (b) H is not a subgroup of G.
(c) H is a subgroup which is not normal in G. (d) $H \subseteq Z(G)$.

12. Let $G = GL_2(\mathbb{R})$, $H = \{A \in G : detA = 2^m 3^n, \text{ for some } m, n \in \mathbb{Z}\}$, then
(a) H is a normal subgroup of G. (b) H is not a subgroup of G.
(c) H is a subgroup which is not normal in G. (d) $H \subseteq Z(G)$.

13. Let $G = U(16)$, $H = \{\bar{1}, \bar{15}\}$, $K = \{\bar{1}, \bar{9}\}$, then

(a) H, K are isomorphic groups and $\dfrac{G}{H}, \dfrac{G}{K}$ are isomorphic groups.

(b) H, K are not isomorphic groups but $\dfrac{G}{H}, \dfrac{G}{K}$ are isomorphic groups.

(c) H is not isomorphic to K.

(d) $\dfrac{G}{H}, \dfrac{G}{K}$ are not isomorphic groups.

14. Let $H = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in 2\mathbb{Z} \right\}, G = M_2(\mathbb{Z})$, under addition of 2×2 matrices. The quotient group $\dfrac{G}{H}$ has
(a) 4 elements (b) 16 elements (c) 12 elements (d) 8 elements

15. Let $G = D_4 = \{e, a, a^2, a^3, b, ab, a^2b, a^3b\}, a^4 = e = b^2, aba = b, H = \{e, b, a^2b, a^2\}, K = \{e, b\}$
(a) K is normal in H and H is normal in G. (b) K is not normal in H.
(c) K is normal in G. (d) H is not normal in G.

16. The quotient group $\left(\dfrac{\mathbb{Q}}{\mathbb{Z}}, +\right)$ is
(a) an infinite group in which only identity is of finite order.
(b) is an infinite cyclic group of finite index.
(c) an infinite group in which every element is of finite order.
(d) None of these.

17. Let G be a non-Abelian group of order pq, where p and q are distinct primes then
(a) $\circ(Z(G)) = p$ (b) $\circ(Z(G)) = q$
(c) $Z(G) = \{e\}$ (d) None of these.

18. If H is any non-trivial sugroup of a cyclic G then G/H

(a) is infinite if G is infinite.

(b) is finite

(c) is not cyclic

(d) None of these.

19. If G be an Abelian group then $H = \{(g, g) : g \in G\}$ is

(a) normal in $G \times G$.

(b) is not normal in $G \times G$.

(c) is not a subgroup of $G \times G$

(d) None of these.

20. The index of centre of a finite non-Abelian group

(a) is $\circ(G)$.

(b) is a prime

(c) can not be a prime

(d) None of these.

21. If N is a normal subgroup of G and all the elements of G/N and N have finite order, then

(a) every element of G has finite order.

(b) every element of G has infinite order.

(c) G can have elements of infinite order.

(d) None of these.

22. If H is a subgroup of S_n having order $n!/2$, then which of the following is not true

(a) H is normal in S_n

(b) $\sigma^2 \in H$ for every $\sigma \in S_n$.

(c) H contains all 3-cycles.

(d) $H \neq A_n$.

23. $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ has
(a) 3 subgroups of order 2. (b) 7 subgroup of order 2 (c) 6 subgroups of order 2.
(d) 9 subgroups of order 2.

24. The order of any non-identity element in $\mathbb{Z}_3 \times \mathbb{Z}_3$ is
(a) 3 (b) 9 (c) 6 (d) none of these.

25. Which of the following statements is false?
(a) $\mathbb{Z}_3 \times \mathbb{Z}_5$ is isomorphic to $\mathbb{Z}_{15}$ (b) $\mathbb{Z}_3 \times \mathbb{Z}_2$ is isomorphic to $\mathbb{Z}_6$
(c) $\mathbb{Z}_9 \times \mathbb{Z}_9$ is isomorphic to $\mathbb{Z}_{27}$ (d) $\mathbb{Z}_4 \times \mathbb{Z}_3$ is isomorphic to $\mathbb{Z}_{12}$

26. The group $S_3 \times \mathbb{Z}_2$ is isomorphic to
(a) $\mathbb{Z}_{12}$ (b) A_4 (c) D_6 (d) $\mathbb{Z}_6 \times \mathbb{Z}_2$

27. Let $G_1 = \mathbb{Z}_4 \times \mathbb{Z}_{15}$ and $G_2 = \mathbb{Z}_6 \times \mathbb{Z}_{10}$, then
   (a) G_1 and G_2 are cyclic groups of order 60.
   (b) G_1 and G_2 are not cyclic groups.
   (c) G_1 is cyclic but G_2 is not cyclic group.
   (d) G_1 is not cyclic but G_2 is a cyclic group.

28. . Which is true about groups?
   (a) $\mathbb{Z}_4 \times \mathbb{Z}_2$ is isomorphic to $V_4 \times \mathbb{Z}_2$.
   (b) $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ is isomorphic to $V_4 \times \mathbb{Z}_2$.
   (c) D_4 (the dihedral group of order 8) is isomorphic to Quaternion group Q_8 of order 8.
   (d) None of these

29. A group of order n is isomorphic to
(a) a subgroup of $\mathbb{Z}_n \times \mathbb{Z}_n$. (b) a subgroup of S_n.
(c) a subgroup of D_n. (d) a subgroup of $\mathbb{Z}_{2n}$

30. $\mathbb{Z}_3$ is isomorphic to the following subgroup of S_3
(a) $<(12)>$. (b) $<(13)>$ (c) A_3 (d) S_3 itself.

31. A group of order 4 in which every element satisfies the equation $x^2 = e$ is isomorphic to
(a) $\mathbb{Z}_2 \times \mathbb{Z}_2$. (b) μ_4, the group of forth roots of unity under multiplication.
(c) $(\mathbb{Z}_4, +)$ (d) $\{\bar{1}, \bar{3}, \bar{7}, \bar{9}\}$.

32. The smallest positive integer n for which there are two non-isomorphic groups of order n equals.
(a) 2 (b) 4 (c) 6 (d) 8

33. For each positive integer n,
(a) There is a cyclic group of order n. (b) There are two non-isomorphic groups of order n.
(c) There is a non-abelian group of order n. (d) The number of non-isomorphic groups of order n is equal to n

34. A non-cyclic group of order 6 is isomorphic to
(a) $\mathbb{Z}_3 \times \mathbb{Z}_2$ (b) μ_6, the group of sixth roots of unity under multiplication.
(c) $U(14) = \{\bar{1}, \bar{3}, \bar{5}, \bar{9}, \bar{11}, \bar{13}\}$. (d) S_3

35. Let $G_1 = \mathbb{Z}_3 \times \mathbb{Z}_5$,$G_2 = \mathbb{Z}_3 \times \mathbb{Z}_9$. Then

(a) G_1 is isomorphic to $\mathbb{Z}_{15}$ and G_2 is isomorphic to $\mathbb{Z}_{27}$.

(b) G_1 and G_2 are not isomorphic to $\mathbb{Z}_{15}$, $\mathbb{Z}_{27}$ respectively.

(c) G_1 is not isomorphic to $\mathbb{Z}_{15}$ but G_2 is isomorphic to $\mathbb{Z}_{27}$

(d) G_1 is isomorphic to $\mathbb{Z}_{15}$ but G_2 is not isomorphic to $\mathbb{Z}_{27}$

36. The number of elements of order 4 in $\mathbb{Z}_8 \times \mathbb{Z}_4$ is
(a) 4 (b) 12 (c) 20 (d) 16

37. Consider the following groups i) $\mathbb{Z}_4$ ii) $U(10)$ ii) $U(8)$ iv) $U(5)$. The only non-isomorphic group among them is
(a) $U(8)$ (b) $U(10)$ (c) $\mathbb{Z}_4$ (d) All are isomorphic.

38. Consider the following groups i) S_3 ii) μ_6 ii) $\mathbb{Z}_6$ iv) $\mathbb{Z}_2 \times \mathbb{Z}_3$ v) $U(9)$. The only non-isomorphic group among them is
(a) S_3 (b) μ_6 (c) $\mathbb{Z}_2 \times \mathbb{Z}_3$ (d) $S_3 \simeq U(9)$ and $\mu_6, \mathbb{Z}_6, \mathbb{Z}_2 \times \mathbb{Z}_3$ are isomorphic. .

39. If for positive integers m, n have $\mathbb{Z}_m \times \mathbb{Z}_n$ is isomorphic to$(\mathbb{Z}_{mn}, +)$ then which is true,

(a) m, n are relatively prime.

(b) m, n are odd.

(c) m, n are prime.

(d) $m = p^r, n = q^s$ for primes p, q and $r, s \in \mathbb{N}$.

40. Let $G = \mathbb{Z}_4 \times \mathbb{Z}_4$ and $H = \mathbb{Z}_4 \times \{\bar{0}, \bar{2}\}, K =< (\bar{2}, \bar{2}) >$ be subgroups of G Then
(a) G/H is isomorphic to K (b) G/H is isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_2$
(c) H and K are isomorphic. (d) none of these.

41. From the given list of pairs of group, pick the pair of non-isomorphic groups
(a) $3\mathbb{Z}/12\mathbb{Z}$ and $\mathbb{Z}_4$ (b) $8\mathbb{Z}/48\mathbb{Z}$ and $\mathbb{Z}_6$
(c) $\mathbb{Z}_4$ and V_4 (d) $(\mathbb{Z} \times \mathbb{Z})/(2\mathbb{Z} \times 2\mathbb{Z})$ and $\mathbb{Z}_2 \times \mathbb{Z}_2$

42. From the given list of pairs of groups, pick the pairs of isomorphic groups
(a) $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ and $\mathbb{Z}_4 \times \mathbb{Z}_2$ (b) $\mathbb{Z}_8$ and $\mathbb{Z}_4 \times \mathbb{Z}_2$
(c) D_4 and $\mathbb{Z}_4 \times \mathbb{Z}_2$ (d) $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ and $V_4 \times \mathbb{Z}_2$

43. If G, H, K are finite Abelian groups and $G \times K \cong H \times K$, then

(a) $G = H$

(b) G need not be isomorphic to H

(c) $G \cong H$

(d) None of these.

44. If G is an Abelian group with order mn where $(m, n) = 1$. $G(m) = \{g \in G : g^m = e\}$ and $G(n) = \{g \in G : g^n = e\}$, then

(a) $G = G(n) \cup G(m)$

(b) $G \cong G(n) \times G(m)$

(c) $G = G(m)G(m)$

(d) None of these.

45. $m, n \in \mathbb{N}$ with $m|n$ and $H = \{\overline{k} \in U(n) : k \equiv 1 \bmod m\}$. Then

(a) $U(n)/H \cong U(m)$

(b) $U(n)/U(m) \cong H$.

(c) $U(m) \cong H$.

(d) None of these.

46. $U(16)/ < \overline{9} >$ is isomorphic to
(a) $\mathbb{Z}_4$ (b) $\mathbb{Z}_2 \times \mathbb{Z}_2$ (c) $\mathbb{Z}_8$ (d) None of these.

47. If $\mathbb{R}^*$ and $\mathbb{R}^+$ are multiplicative groups and $f : \mathbb{R}^* \to \mathbb{R}^+$ is defined by $f(x) = |x|$ then

(a) f is an injective homomorphism.

(b) f is not a homomorphism.

(c) $\ker f = \{-1, 1\}$.

(d) f is an isomorphism.

Ring, Subring, Ideal and Integral domain, Homomorphism, Isomorphism of Rings

48. Let R be a ring and a, b be non-zero elements of R. The equation $ax = b$

    (a) has a unique solution in R.
    (c) has atmost one solution in R.
    (b) may have more than one solution in R.
    (d) None of these.

49. The group of units of the ring $\mathbb{Z}_{25}$ is

    (a) $\{\bar{1}, \bar{3}, \bar{5}, \bar{7}\}$ mod 25.
    (b) $\{\bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{6}, \bar{7}, \bar{8}, \bar{9}, \bar{11}, \bar{12}, \bar{13}, \bar{14}, \bar{16}, \bar{17}, \bar{18}, \bar{19}, \bar{21}, \bar{22}, \bar{23}, \bar{24}\}$ mod 25.
    (c) $\{\bar{1}, \bar{4}, \bar{8}, \bar{12}, \bar{16}, \bar{20}\}$ mod 25.
    (d) $\{\bar{1}, \bar{3}, \bar{6}, \bar{9}, \bar{12}, \bar{15}, \bar{18}, \bar{21}, \bar{24}\}$ mod 25.

50. The group of units of a ring is
    (a) abelian but may not be cyclic (b) Cyclic (c) may not be abelian (d) finite

51. Consider the ring $M_2(\mathbb{Z}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{Z} \right\}$ under addition and multiplication of 2×2 matrices, then $A \in M_2(\mathbb{Z})$ is an unit if -

    (a) $ad - bc \neq 0$
    (c) and only if $ad - bc \neq 0$.
    (b) $ad - bc$ is an even integer.
    (d) $ad - bc = \pm 1$.

52. Consider the following rings :
    (i) $(\mathbb{Z}_5, +, \cdot)$ (ii) $(\mathbb{Z}_{15}, +, \cdot)$ (iii) $\mathbb{Z} \times \mathbb{Z}$ under component wise addition and multiplication (iv) $\mathbb{R}[x]$,Then

    (a) (i), (iv) have no proper zero divisors.
    (b) (i), (iii) have no proper zero divisors
    (c) (i), (iii) have proper zero divisors.
    (d) (i), (iii), (iv) have no proper zero divisors

53. The number of units in the ring $\mathbb{Z}_{20}$ is
    (a) 5 (b) 6 (c) 7 (d) 8

54. Which of the following is a subring of $(\mathbb{Q}, +, \cdot)$

    (i) $R = \{a/b \,/\, a, b \in \mathbb{Z},\ (a, b) = 1, b \neq 0, b$ is not divisible by $3\}$.
    (ii) $R = \{a/b \,/\, a, b \in \mathbb{Z},\ (a, b) = 1, b \neq 0, b$ is divisible by $3\}$.
    (iii) $R = \{a/b \,/\, a, b \in \mathbb{Z},\ (a, b) = 1, b \neq 0, a$ is divisible by $3\}$.
    (iii) $R = \{x^2 : x \in \mathbb{Q}\}$.

    (a) (i) and (iv) (b) (ii) and (iv) (c) (i) and (ii) (d) only (i).

55. Let R and S be rings. Consider the ring $R \times S$ under component wise addition and multiplication.
(a) If R, S are integral domains, then $R \times S$ is an integral domain.
(b) $R \times S$ is an integral domain if and only if R, S are integral domains.
(c) $R \times S$ is not an integral domain, whatever R, S may be.
(d) $R \times S$ is not commutative even if R, S are commutative.

56. Let R be an integral domain. Then, $x^2 = 1$
(a) has exactly two solutions. (b) may not have any solution.
(c) may have more than two solutions. (d) None of these.

57. Consider the following rings: (i) $\mathbb{Z}_{18}$ (ii) $\mathbb{Z}_{12}$ (iii) $\mathbb{Z}_{10}$ (iv) $\mathbb{Z}_{14}$, then
(a) (i), (ii), (iii) ,(iv) have nilpotent elements. (b) (i), (ii) have nilpotent elements.
(c) (iii), (iv) have nilpotent elements. (d) None of these have nilpotent elements.

58. In an integral domain the number of elements which are their own inverses is
(a) 1 (b) 1 or 2 (c) 2 (d) infinitely many.

59. In a ring $(\mathbb{Z}_n, +, \cdot)$ where n is a positive integer > 1

(i) $\bar{a}^2 = \bar{a} \Rightarrow \bar{a} = 0$ or $\bar{a} = \bar{1}$ for $\bar{a} \in \mathbb{Z}_n$.
(ii) $\bar{a} \cdot \bar{b} = \bar{0} \Rightarrow \bar{a} = \bar{0}$ or $\bar{b} = \bar{0}$ for $\bar{a}, \bar{b} \in \mathbb{Z}_n$.
(iii) $\bar{a} \cdot \bar{b} = \bar{a} \cdot \bar{c}$, $\bar{a} \neq 0 \Rightarrow \bar{b} = \bar{c}$ for $\bar{b}, \bar{c} \in \mathbb{Z}_n$. Then,

(a) the statements (i), (ii), (iii) are true.
(b) the statements (i) is true but (ii), (iii) may not be true.
(c) the statements (i), (ii), (iii) are true if n is prime.
(d) None of the above.

60. If R is a ring and a, b are zero divisors in R, then
(a) $a + b$ is always a zero divisor.
(c) $a + b$ may not be a zero divisor.
(b) $a + b$ is not a unit in R.
(d) None of these.

61. In the ring $R = \left\{ \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} : a, b, d \in \mathbb{Z}_2 \right\}$, the number of non-zero zero divisors is
(a) 6 (b) 7 (c) 5 (d) None of these.

62. If x is an idempotent element in $\mathbb{Z}_n$ $(x^2 = x)$, then
(a) $1 - x$ is a unit. (b) $1 + x$ is a unit.
(c) $1 - x$ is an idempotent. (d) None of these.

63. Let R be a commutative ring such that $a^2 = 0 \Rightarrow a = 0 \; \forall \, a \in R$, then
(a) R has no proper zero divisors. (b) R has no nilpotent elements.
(c) R is an integral domain but not a field. (d) None of these.

64. Consider the rings $R_1 = (\mathbb{Z}_{10}, +, \cdot), R_2 = (\mathbb{Z}_{23}, +, \cdot), R_3 = M_2(\mathbb{Z})), R_4 = \mathbb{Z} \times \mathbb{Z}$ under component wise addition and multiplication.
(a) R_1, R_2, R_3, R_4 are all integral domains. (b) Only R_2, R_3, R_4 are integral domains.
(c) R_2 is an integral domain. (d) R_2, R_4 are integral domains.

65. Let R be an integral domain of characteristic p. Then,
(a) $(x+y)^m = x^m + y^m \ \ \forall x, y \in R$ if and only if $m = p$.
(b) $(x+y)^m = x^m + y^m \ \ \forall x, y \in R$ and $m = kp$.
(c) $(x+y)^{p^n} = x^{p^n} + y^{p^n} \ \ \forall x, y \in R$ and for all $n \in \mathbb{N}$.
(d) None of the above.

66. Consider the subset $S = \{\bar{0}, \bar{2}, \bar{4}, \bar{6}, \bar{8}\}$ of $\mathbb{Z}_{10}$.
(a) S is a subring of $\mathbb{Z}_{10}$. (b) S is not a subring of $\mathbb{Z}_{10}$.
(c) S is a subring with multiplicative identity $\bar{6}$.
(d) S is a ring with multiplicative identity $\bar{6}$.

67. Let R be a ring in which $x^2 = x$ for all $x \in R$. Then,
(a) R is an integral domain with characteristic 3.
(b) R is field with characteristic 3.
(c) Characteristic of R is 2. (d) None of these.

68. The characteristics of the ring $\mathbb{Z}_{12} \times \mathbb{Z}_{15}$ under component wise addition and multiplication is
(a) 180 (b) 3 (c) 60 (d) 6

69. $x \in \mathbb{R}[x]$ is
(a) is a unit in $\mathbb{R}[x]$.
(b) is a zero divisor in $\mathbb{R}[x]$.
(c) is neither a unit nor a zero divisor in $\mathbb{R}[x]$.
(d) None of these.

70. If $E_{1,1} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and $R = M_{2\times 2}(\mathbb{R})$ then $RE_{1,1}$ is
(a) is a subring with multiplicative identity I_2.
(b) is a subring with multiplicative identity $E_{1,1}$.
(c) is not a subring of R.
(d) None of these.

71. Which of the following is true
(a) $\mathbb{Z}_2[i], \mathbb{Z}_5[i]$ are integral domains and $\mathbb{Z}_3[i]$ is a field.
(b) $\mathbb{Z}_2[i], \mathbb{Z}_5[i]$ and $\mathbb{Z}_3[i]$ are fields
(c) $\mathbb{Z}_2[i], \mathbb{Z}_5[i]$ are fields and $\mathbb{Z}_3[i]$ is an integral domain.
(d) Only $\mathbb{Z}_2[i]$ is a field and $\mathbb{Z}_3[i], \mathbb{Z}_5[i]$ are integral domains.

72. If $H_{\mathbb{Z}} = \{a + bi + cj + dk : a, b, c, d \in \mathbb{Z}\}$ then the multiplicative group of units of $H_{\mathbb{Z}}$ is
(a) $\{\pm 1\}$.
(b) $\{1, i, j, k\}$
(c) $\{\pm 1, \pm i, \pm j, \pm k\}$
(d) $H_{\mathbb{Z}} - \{0\}$.

73. Consider the ring $\mathbb{Z} \times \mathbb{Z}$ under component wise addition and multiplication.
Let $I = \{(a, -a) \ : \ a \in \mathbb{Z}\}, J = \{(a, 0) \ : \ a \in \mathbb{Z}\}$. Then,
(a) I and J are ideals of $\mathbb{Z} \times \mathbb{Z}$.
(b) I is an ideal of $\mathbb{Z} \times \mathbb{Z}$ but J is not an ideal of $\mathbb{Z} \times \mathbb{Z}$.
(c) Neither I nor J is an ideal of $\mathbb{Z} \times \mathbb{Z}$.
(d) J is an ideal of $\mathbb{Z} \times \mathbb{Z}$ but I is not an ideal of $\mathbb{Z} \times \mathbb{Z}$.

74. Consider the ring $M_2(\mathbb{Z}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{Z} \right\}$ and
let $I = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \text{ are divisible by } 5 \right\}$. Then

(a) I is a subring of $M_2(\mathbb{Z})$ but not an ideal.

(b) I is an ideal of $M_2(\mathbb{Z})$ but not a subring.

(c) I is not an ideal of $M_2(\mathbb{Z})$.

(d) I is both a subring and an ideal of $M_2(\mathbb{Z})$.

75. Consider the ideal $I = 10\mathbb{Z}$ and $J = 12\mathbb{Z}$, then
(a) $I + J = 22\mathbb{Z}$, $IJ = 120\mathbb{Z}$. (b) $I + J = 2\mathbb{Z}$, $IJ = 60\mathbb{Z}$.
(c) $I + J = 2\mathbb{Z}$, $IJ = 120\mathbb{Z}$. (d) None of these.

76. In the ring of integers $\mathbb{Z}$, consider the ideals $I = 4\mathbb{Z} + 6\mathbb{Z}$, $J = m\mathbb{Z} + n\mathbb{Z}$, $m, n \in \mathbb{N}$. Then,
(a) $I = 24\mathbb{Z}, J = mn\mathbb{Z}$ (b) $I = 2\mathbb{Z}, J = d\mathbb{Z}$ where $d = \ gcd\ (m, n)$
(c) $I = 12\mathbb{Z}, J = l\mathbb{Z}$, where $l = \ lcm\ (m, n)$ (d) None of the above.

77. In the ring of integers $\mathbb{Z}$, consider the ideal $I = (6\mathbb{Z})(4\mathbb{Z})$, then
(a) $I = 24\mathbb{Z}$. (b) $I = 12\mathbb{Z}$. (c) $I = 2\mathbb{Z}$. (d) None of these.

78. $(n\mathbb{Z})(m\mathbb{Z}) = (n\mathbb{Z}) \cap (m\mathbb{Z})$ if and only if
(a) $m|n$. (b) $(m, n) = 1$. (c) $m = n$. (d) None of these.

79. Which of the following is true
(i) $\mathbb{Z}$ is an ideal of $\mathbb{Q}$ (ii) $\{(n, n) : n \in \mathbb{Z}\}$ is an ideal of $\mathbb{Z} \times \mathbb{Z}$
(iii) $\{f \in F(\mathbb{R}, \mathbb{R}) : f(\pi) = 0\}$ is an ideal in the ring $F(\mathbb{R}, \mathbb{R})$ of real valued functions.
(a) (i), (ii) (b) (ii), (iii) (c) only (ii). (d) None of these.

80. The number of ring homomorphisms from $\mathbb{Q}$ to itself is
(a) 1 (b) 2 (c) infinitely many (d) none of these.

81. The number of ring homomorphisms from $\mathbb{C}$ to itself is
(a) 1 (b) 2 (c) infinitely many (d) none of these.

82. Consider the following pair of rings.
(i) $\mathbb{Z}[\sqrt{2}]$ and $\mathbb{Z}[\sqrt{5}]$ (ii) $\mathbb{Z}[\sqrt{-2}]$ and $\mathbb{Z}[\sqrt{-5}]$ (iii) $\mathbb{Q}$ and $\mathbb{R}$ (iv) $M = \left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} : a, b \in \mathbb{R} \right\}$,$\mathbb{C}$

(a) (i) and (iv) are isomorphic pairs of rings.

(b) (i) and (ii) are isomorphic pairs of rings.

(c) only (iv) is an isomorphic pair of rings.

(d) (i), (ii) and (iv) are isomorphic pairs of rings.

83. Consider the following maps from $M_2(\mathbb{Z}_p) \to \mathbb{Z}_p$ defined by
$f\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right) = a$, $g\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right) = a + d$, $h(A) = det\, A$ for $A \in M_2(\mathbb{Z}_p)$.
(a) f, g, h are all ring homomorphisms. (b) only h is a ring homomorphism.
(c) f is a ring homomorphism and g, h are not. (d) none of these.

84. Consider the map $\pi_1 : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ and $\pi_2 : \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ defined by $\pi_1(m, n) = m$, $\pi_2(m) = (m, 0)$, where $\mathbb{Z} \times \mathbb{Z}$ denotes ring with component wise addition and multiplication.
(a) π_1 and π_2 are ring homomorphisms.
(b) Both π_1, π_2 are not ring homomorphisms.
(c) π_1 is a ring homomorphism but π_2 is not a ring homomorphism.
(d) π_2 is a ring homomorphism but π_1 is not a ring homomorphism.

85. The number of ring homomorphisms from $\mathbb{Z}$ to $\mathbb{Z}$ are
(a) one (b) zero (c) two (d) infinitely many.

86. Let $\phi_n : \mathbb{Z}[x] \to \mathbb{Z}_n[x]$ be defined by $\phi(a_0 + a_1x + \cdots a_kx^k) = \bar{a_0} + \bar{a_1}x + \cdots \bar{a_k}x^k$, is a ring homomorphism only if
(a) n is a prime number. (b) n is a positive integer.
(c) n is an odd integer. (d) n is an even integer.

87. The kernel of the ring homomorphism $\phi : \mathbb{R}[x] \to \mathbb{C}$ defined by $\phi(f(x)) = f(2 + i)$ is
(a) $\{f(x) \in \mathbb{R}[x] : (x - 2)|f(x)\}$. (b) $\{f(x) \in \mathbb{R}[x] : (x^2 - 4x - 5)|f(x)\}$.
(c) $\{f(x) \in \mathbb{R}[x] : (x^2 - 4x + 2)|f(x)\}$. (d) $\{f(x) \in \mathbb{R}[x] : (x^2 - 4x + 5)|f(x)\}$

88. Consider the ring homomorphism $\phi : \mathbb{R}[x] \to \mathbb{R}$ defined by $\phi(a_0 + a_1x + \cdots + a_nx^n) = a_0 + a_1 + \cdots + a_n$. Then, the kernel ϕ is
(a) $\{f(x) \in \mathbb{R}[x] : f(1) = 1\}$. (b) $\{f(x) \in \mathbb{R}[x] : f(1) = 0\}$.
(c) $\{f(x) \in \mathbb{R}[x] : f(0) = 1\}$. (d) None of these.

89. Consider the ring homomorphism $\phi : \mathbb{R}[x] \to \mathbb{R}$ defined by $\phi(a_0 + a_1x + \cdots + a_nx^n) = \sum_{k=0}^{n}(-1)^k a_k$. Then, the kernel ϕ is
(a) $\{f(x) \in \mathbb{R}[x] : f(-1) = 0\}$. (b) $\{f(x) \in \mathbb{R}[x] : f(-1) = 1\}$.
(c) $\{f(x) \in \mathbb{R}[x] : f(-1) = -1\}$. (d) None of these.

90. Ring $H = \left\{\begin{pmatrix} a & 2b \\ b & a \end{pmatrix} : a, b \in \mathbb{Z}\right\}$ is

(a) is not isomorphic to $\mathbb{Z}[\sqrt{2}]$.
(b) is isomorphic to $\mathbb{Z}[\sqrt{2}]$.
(c) is isomorphic to $\mathbb{Q}[\sqrt{2}]$.
(d) None of these.

91. Consider the maps $\phi_1 : M_2(\mathbb{Z}) \to \mathbb{Z}$ defined by $\phi_1\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right) = a$ and $\phi_2 : R \to \mathbb{Z}$ defined by $\phi_1\left(\begin{pmatrix} a & b \\ 0 & d \end{pmatrix}\right) = a$, where $R = \left\{\begin{pmatrix} a & b \\ 0 & d \end{pmatrix} : a, b, d \in \mathbb{Z}\right\}$. Then

(a) ϕ_1 is a ring homomorphism but ϕ_2 is not a ring homomorphism.
(b) Both ϕ_1, ϕ_2 are ring homomorphisms.
(c) ϕ_2 is a ring homomorphism but ϕ_1 is not a ring homomorphism.
(d) Both ϕ_1, ϕ_2 are not ring homomorphisms.

92. The factor ring $\dfrac{\mathbb{Z}[i]}{(1+i)}$ is
(a) is an infinite ring. (b) a field having 2 elements.
(c) a ring having 4 elements. (d) a ring with proper zero-divisors.

Divisibility, Prime ideals, Maximal ideals, Polynomial ring and Field

93. Let $R = M_2(\mathbb{Z})$ and $I = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{Z}, \text{ and are divisible by } 5 \right\}$
   (a) I is not an ideal. (b) I is a prime ideal but not a maximal ideal.
   (c) I is a maximal ideal. (d) I is an ideal but not a prime ideal.

94. Let R be a commutative ring. If (0) is the only maximal ideal in R, then
   (a) R is finite ring. (b) R is an integral domain, but not field.
   (c) R is a field. (d) None of the above.

95. The number of maximal ideals in $\mathbb{Z}_{16}$ are
   (a) 1. (b) 2. (c) 3. (d) 4.

96. Let $R = M_2(\mathbb{Z}_2)$ and $I = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} A : A \in R \right\}$. Then
   (a) I is not an ideal. (b) I is a prime ideal but not a maximal ideal.
   (c) I is a maximal ideal. (d) I is an ideal but not a prime ideal.

97. Let $R = C[0, 1]$, the ring of continuous real valued functions on [0,1] under pointwise addition and multiplication, $I = \{f \in R : f(1/2) = 0\}$.
   (a) I is not an ideal. (b) I is a prime ideal but not a maximal ideal.
   (c) I is a maximal ideal. (d) I is an ideal but not a prime ideal.

98. In the polynomial ring $\mathbb{Z}[x]$, consider $I = \{f(x) : f(0) = 0\}$, then
   (a) I is an ideal. (b) I is prime ideal but not maximal ideal.
   (c) I is a maximal ideal. (d) I is ideal but neither prime ideal nor maximal.

99. If R is an infinite integral domain and I is a proper ideal then

   (a) R/I is an integral domain. (b) R/I is a field.
   (c) R/I is an infinite ring. (d) R/I is commutative.

100. Let R be a finite commutative ring. Then
   (a) R is a field. (b) (0) is the only proper ideal of R.
   (c) every prime ideal is maximal. (d) R is an integral domain.

101. Let $S = \{a + ib : a, b \in \mathbb{Z}, \text{ are divisible by } 5\ \}$. Then,
   (a) S is not an ideal but is a subring of $\mathbb{Z}[i]$.
   (b) S is an ideal as well as subring of $\mathbb{Z}[i]$.
   (c) S is an ideal of $\mathbb{Z}[i]$.
   (d) None of these.

102. Let R be a commutative ring, and P_1 and P_2 are prime ideals of R, then
   (a) $P_1 \cup P_2$ and $P_1 \cap P_2$ both are prime ideals of R.
   (b) $P_1 \cap P_2$ is prime ideal of R always but $P_1 \cup P_2$ may not be.
   (c) If $P_1 \subseteq P_2$ or $P_2 \subseteq P_1$ then $P_1 \cap P_2$ is prime ideal of R.
   (d) None of the above.

103. Which of the following is irreducible in $\mathbb{Z}[\sqrt{5}]$
(a) $9+4\sqrt{5}$ (b) $1+\sqrt{5}$ (c) 5 (d) $7+3\sqrt{5}$

104. In the ring $\mathbb{Q}[x]$, the principal ideal (x^2+bx+c) is a maximal ideal if
(a) $b=c=0$ (b) b^2-4c is not a square of a rational number.
(c) b^2-4c is a square of a rational number. (d) b^2-4c is an integer.

105. In the ring $\mathbb{Z}[x]$,

(a) (x) is a maximal ideal.

(b) (x) is a prime ideal which is not maximal.

(c) there is no maximal ideal in $\mathbb{Z}[x]$.

(d) (x) is not a prime ideal.

106. In the ring $\mathbb{Z}[\sqrt{5}]$
(a) $1+\sqrt{5}$ is irreducible but not prime. (b) $1+\sqrt{5}$ is prime
(c) $1+\sqrt{5}$ is not irreducible (d) $1+\sqrt{5}$ is a unit.

107. In the ring $\mathbb{Z}[\sqrt{-5}]$
(a) $1+\sqrt{-5}$ is not irreducible (b) $1+\sqrt{-5}$ is prime
(c) $1+\sqrt{-5}$ is irreducible but not prime (d) $1+\sqrt{-5}$ is a unit.

108. Consider the following pairs of elements in the given rings respectively. (i) $2+i$ and $1-2i$ in $\mathbb{Z}[i]$ (ii) $1-\sqrt{-5}$ and $7-3\sqrt{-5}$ in $\mathbb{Z}[\sqrt{-5}]$ (iii) 2 and $1+i$ in $\mathbb{Z}[i]$. Then
(a) (i) and (iii) are pairs of associates (b) (i) and (ii) are pairs of associates
(c) (i), (ii) and (iii) are pairs of associates. (d) only (i) is a pair of associates.

109. Consider the following elements in $\mathbb{Z}[\sqrt{-5}]$ (i) $6+\sqrt{-5}$ (ii) 7 (iii) $2-3\sqrt{-5}$. Then
(a) (ii) and (iii) are irreducible and (i) is not irreducible
(b) (i) and (iii) are irreducible and (ii) is not irreducible
(c) (i),(ii) and (iii) are all irreducible (d) (i),(ii) and (iii) are all reducible.

110. Which of the following is true in $\mathbb{Z}[\sqrt{-5}]$
(a) $2+\sqrt{-5}$ is irreducible but not prime. (b) $2+\sqrt{-5}$ is prime.
(c) 3 is prime. (d) $2+\sqrt{-5}$ is reducible.

111. The number of maximal ideals in $\mathbb{R}\times\mathbb{R}\times\mathbb{R}$ is
(a) 1. (b) 3. (c) 6. (d) 9.

112. Which of the following is prime in $\mathbb{Z}[i]$,
(a) 2. (b) 5. (c) 17. (d) 3.

113. A prime integer p is irreducible in $\mathbb{Z}[i]$ if
(a) p is of the form $4k+1$. (b) p is of the form $4k+3$.
(c) p is an odd prime. (d) None of these.

114. The quotient ring $\dfrac{\mathbb{Z}[i]}{(1+i)}$

(a) an integral domain which is not a field.

(b) a field having 2 elements.

(c) a field having 4 elements.

(d) a ring with proper zero divisors.

115. The ring $\dfrac{\mathbb{R}[x]}{(x^4+1)}$ is
(a) an infinite integral domain. (b) an infinite field.
(c) a finite field. (d) None of these.

116. Consider the ring homomorphisms $f_1 : \mathbb{Z}[i] \to \mathbb{Z}_2$ defined by $f(a+bi) = (a-b)\, mod\, 2$ and $f_2 : \mathbb{Z}[i] \to \mathbb{Z}_5$ defined by $f(a+bi) = (a-2b)\, mod\, 5$. Then

(a) ker f_1 is a maximal ideal but ker f_2 is not a maximal ideal.

(b) ker f_2 is a maximal ideal but ker f_1 is not a maximal ideal.

(c) both ker f_1 and ker f_2 are not maximal ideals.

(d) both ker f_1 and ker f_2 are maximal ideals.

117. In the ring $\mathbb{R}[x]$ and $\mathbb{C}[x]$, consider the ideal $I = (x^2 - x + 2)$

(a) I is a maximal ideal in both $\mathbb{R}[x]$ and $\mathbb{C}[x]$.

(b) I is a maximal ideal in $\mathbb{R}[x]$ but not $\mathbb{C}[x]$.

(c) I is a maximal ideal in $\mathbb{C}[x]$ but not in $\mathbb{R}[x]$.

(d) I is a not maximal ideal in both $\mathbb{R}[x]$ and $\mathbb{C}[x]$.

118. If $R_1 = \mathbb{Z}[\sqrt{2}] = \{a + b\sqrt{2} : a, b \in \mathbb{Z}\}, R_2 = \mathbb{Z}[\sqrt{5}] = \{a + b\sqrt{5} : a, b \in \mathbb{Z}\}, R_3 = \mathbb{Q}[\sqrt{2}] = \{a + b\sqrt{2} : a, b \in \mathbb{Q}\}, R_4 = \mathbb{Z}[i] = \{a + bi : a, b \in \mathbb{Z}\}$, then
(a) R_1, R_2, R_3, R_4 are integral domains which are not fields.
(b) R_1, R_2, R_4 are integral domains which are not fields and R_3 is a field.
(c) R_1, R_2, R_3, R_4 are all fields.
(d) None of the above.

119. Consider the ring $S = \left\{ \begin{pmatrix} a & a \\ a & a \end{pmatrix} \;\; a \in \mathbb{Q} \right\}$
(a) S is an integral domain which is not a field.
(b) S is a field with multiplicative identity $\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$.
(c) S is a non-commutative ring. (d) None of these.

120. Let R and S be rings. Consider the ring $R \times S$ under component wise addition and multiplication.
(a) If R, S are fields, then $R \times S$ is a field.
(b) If R, S are integral domains, then $R \times S$ is an integral domain.
(c) $R \times S$ is not a field, whatever R, S may be.
(d) None of the above.

121. Let F_1 and F_2 be fields having 9 and 16 elements respectively. Then, the number of (non-trivial) ring homomorphism from F_1 to F_2 are
(a) One (b) zero (c) two (d) None of the above.

122. Consider the rings $R_1 = (\mathbb{Z}_{10}, +, \cdot), R_2 = (\mathbb{Z}_{17}, +, \cdot), R_3 = \{\frac{a}{b} : a, b \in \mathbb{Z},\, b \text{ is odd }\}$.
(a) R_1, R_2, R_3 are all fields. (b) Only R_1, R_2 are fields.
(c) Only R_2, R_3 are fields. (d) Only R_2 is a field.

123. Let $R = \mathbb{R}[x]/(2x)$. Then,
(a) R is a field. (b) R is an integral domain but not a field.
(c) R is not an integral domain. (d) R is a finite commutative ring.

124. There exist fields of
(a) 10 elements. (b) 7,8,9 elements. (c) 12 elements. (d) 6 elements.

125. The field of quotients of $\mathbb{Z}[\sqrt{2}]$ is
(a) $\mathbb{Q}[\sqrt{2}]$ (b) $\mathbb{R}$ (c) $\mathbb{Q}$ (d) $\mathbb{C}$.

126. The field of quotients of $\mathbb{Z}[i]$ is
(a) $\mathbb{Q}[i]$ (b) $\mathbb{R}$ (c) $\mathbb{C}$ (d) None of these.

127. Which of the following statements is true?

(a) R is integral domain $\Rightarrow$ $R[x]$ is an integral domain.

(b) R is a division ring $\Rightarrow$ $R[x]$ is division ring.

(c) R is field $\Rightarrow$ $R[x]$ is field.

(d) None of these.

128. The polynomial $f(x) = 2x^2 + 4$ is reducible over
(a) $\mathbb{Z}$ (b) $\mathbb{Q}$ (c) $\mathbb{R}$ (d) None.

129. Which of the following polynomials in $\mathbb{Z}[x]$ satisfy an Eisenstein criterion for irreducibility in $\mathbb{Q}$.
(i) $x^2 - 12$ (ii) $8x^3 + 6x^2 - 9x + 24$
(iii) $4x^{10} - 9x^3 + 24x - 18$ (iv) $2x^{10} - 25x^3 + 10x^2 - 30$
(a) All. (b) (ii) and (iii).
(c) (i), (ii) and (iv). (d) only (i).

130. The polynomial $8x^3 - 6x + 1$ is
(a) reducible over $\mathbb{Z}$ (b) is reducible over $\mathbb{Q}$
(c) is irreducible over $\mathbb{Q}$ (d) is irreducible over $\mathbb{R}$.

131. Let $f(x) = x^2 - 2$, then

(a) $f(x)$ is reducible in $\mathbb{Q}[x]$.

(b) $f(x)$ is irreducible in $\mathbb{Q}[x]$ but reducible in $\mathbb{Q}[\sqrt{2}][x]$.

(c) $f(x)$ is irreducible in $\mathbb{R}[x]$

(d) None of these.

132. Let $f(x) = x^2 - 2$, then

(a) $f(x)$ is reducible in $\mathbb{Z}_3[x]$ and $\mathbb{Z}_5[x]$.

(b) $f(x)$ is irreducible in $\mathbb{Z}_3[x]$ but reducible in $\mathbb{Z}_5[x]$.

(c) $f(x)$ is reducible in $\mathbb{Z}_3[x]$ but irreducible in $\mathbb{Z}_5[x]$.

(d) $f(x)$ is irreducible in both $\mathbb{Z}_3[x]$ and $\mathbb{Z}_5[x]$.

133. Let R be a commutative ring and $f(x)$ be a polynomial of degree n over R. Then the no. of roots of $f(x)$ in R is
(a) less than or equal to n. (b) equal to n
(c) strictly less than n (d) may be greater than n.

134. The polynomial $2x + 1$ is
(a) unit in $\mathbb{Z}_8[x]$ (b) zero divisor in $\mathbb{Z}_8[x]$ but not nilpotent.
(c) nilpotent in $\mathbb{Z}_8[x]$ (d) None of the above

135. Let $f(x) \in \mathbb{Z}[x]$. Which of the following is true?
(a) If $f(x)$ is reducible over $\mathbb{Q}$, then it is reducible over $\mathbb{Z}$.
(b) $f(x)$ is reducible over $\mathbb{Q}$, but it may not be reducible over $\mathbb{Z}$.
(c) $f(x)$ is reducible over $\mathbb{Q}$.
(d) none of these.

136. Let $I = (x^2 + x + 1)$ in $\mathbb{Z}_n[x]$, $1 \leq n \leq 10$ Then, $\mathbb{Z}_n[x]/I$ is a field if
(a) $n = 3$ (b) $n = 2, 5$ (c) $n = 7$ (d) None of these.

137. The polynomial x is irreducible in $\mathbb{Z}_n[x]$
(a) for each n (b) for $n \geq 3$ (c) iff n is prime (d) never.

138. The number of roots of the polynomial $x^{25} - 1$ in $\mathbb{Z}_{37}[x]$ is
(a) 25 (b) 5 (c) 24 (d) 1

139. Let $f(x) = x^3 - x^2 + 1$
(a) $(f(x))$ is a maximal ideal in $\mathbb{Z}_2[x], \mathbb{Z}_3[x]$ and $\mathbb{Z}_5[x]$.
(b) $(f(x))$ is a maximal ideal in $\mathbb{Z}_3[x]$ and $\mathbb{Z}_5[x]$ but not in $\mathbb{Z}_2[x]$.
(c) $(f(x))$ is a maximal ideal in $\mathbb{Z}_2[x]$ and $\mathbb{Z}_3[x]$ but not in $\mathbb{Z}_5[x]$.
(d) None of the above

140. Let $f(x) = x^{10} + x^9 + \cdots + x + 1, g(x) = x^{11} + x^{10} + x^9 + \cdots + x + 1$. Then
(a) $f(x), g(x)$ are both irreducible over $\mathbb{Z}[x]$.
(b) $f(x), g(x)$ are both reducible over $\mathbb{Z}[x]$.
(c) $f(x)$ is irreducible over $\mathbb{Z}[x]$, $g(x)$ is not.
(d) $g(x)$ is irreducible over $\mathbb{Z}[x]$, $f(x)$ is not.

141. The polynomial $f(x) = x$ is
(a) irreducible over any ring R. (b) irreducible but not prime over any ring R.
(c) can be factored in some polynomial ring. (d) has no roots.

142. In $\mathbb{R}[x]$, Let $I = \{f(x) \in \mathbb{R}[x] : f(2) = f'(2) = f''(2) = 0\}$ and $J = \{f(x) \in \mathbb{R}[x] : f(2) = 0, f'(3) = 0\}$
(a) I, J are ideals in $\mathbb{R}[x]$. (b) I is an ideal, J is not.
(c) Neither I nor J is an ideal. (d) I is a prime ideal in $\mathbb{R}[x]$.