

QUESTION BANK FOR SEMESTER V ATKT
TYBSC MATHS PAPER III (TOPOLOGY OF METRIC SPACE)

	Choose correct alternative in each of the following			
1	Let $(X, \ \cdot \)$ be a normed linear space and $x, y, z \in X$. If d is metric induced by the norm then			
	(a)	$d(x+z, y+z) \geq d(x, y)$	(b)	$d(x+z, y+z) \geq d(x, y) + d(y, z)$
	(c)	$d(x+z, y+z) = d(x, y)$	(d)	None of these.
2	The usual distance on R is given as follows.			
	(a)	$d(x, y) = x - y $	(b)	$d(x, y) = \ x - y\ $
	(c)	$d(x, y) = x - 2y $	(d)	None of these.
3	Let $(X, \ \cdot \)$ be a normed linear space and $x, y \in X$. Then			
	(a)	$\ x - y\ \leq \ x\ - \ y\ $	(b)	$\ x - y\ = \ x\ - \ y\ $
	(c)	$\ x - y\ \geq \ x\ - \ y\ $	(d)	None of these.
4	Let (X, d) be a metric space and $x, y, z \in X$. Then the triangle inequality is given as.....			
	(a)	$d(x, y) \leq d(x, z) + d(z, y)$	(b)	$d(x, y) \geq d(x, z) + d(z, y)$
	(c)	$d(x, y) = d(x, z) + d(z, y)$	(d)	$d(x, y) > d(x, z) + d(z, y)$
5	Let (X, d) be a metric space then which of the following is an induced metric.			
	(a)	$d_1(x, y) = \sqrt{d(x, y)}$	(b)	$d_1(x, y) = d^2(x, y)$
	(c)	$d_1(x, y) = \max\{1, d(x, y)\}$	(d)	None of these.
6	Let (X, d) be a metric space and $x, y \in X$. Let $d(x, y) = s > 0$. Then $B(x, r) \cap B(y, r) = \emptyset$ if..			
	(a)	$r \geq \frac{s}{2}$	(b)	$0 < r \leq \frac{s}{2}$
	(c)	$r \geq 2s$	(d)	None of these.
7	Let (X, d) be a discrete metric space and $x \in X$. Then which of the following open ball is not a singleton set.			
	(a)	$B(x, \frac{1}{2})$	(b)	$B(x, \frac{3}{4})$
	(c)	$B(x, 1)$	(d)	$B(x, r), r > 1$
8	Let (X, d) be a metric space in which the only open subsets are \emptyset and X . Then			
	(a)	d is discrete metric on X .	(b)	$d(x, y) \geq 1$, if $x \neq y$.
	(c)	X is a singleton set.	(d)	None of these.
9	The set $U = \{(x, y) \in R^2 / x^2 - y^2 \leq 1\}$ with Euclidean metric is....			
	(a)	An Open set in R^2 .	(b)	A Closed set in R^2 .
	(c)	Both Open and closed set in R^2 .	(d)	None of these.
10	A rectangle of the form $(a, b) \times (c, d)$ is an open set in			
	(a)	R^2 with Euclidean metric.	(b)	R^2 with $\ \cdot \ _2$ norm.
	(c)	R^2 with discrete metric.	(d)	All of the above.
11	The set of rational numbers Q is.....			

	(a)	An open set in R with usual metric.	(b)	A closed set in R with usual metric.
	(c)	Neither open nor closed in R with usual metric.	(d)	None of these.
12	In a metric space (X, d)			
	(a)	An arbitrary intersection of open set is an open set.	(b)	An arbitrary intersection of open ball is an open ball.
	(c)	An intersection of finitely many open balls is an open ball.	(d)	None of these.
13	The set $U = R \setminus Z$, subset of R with usual metric, is ..			
	(a)	An open set in R .	(b)	A closed set in R .
	(c)	Neither open nor closed in R .	(d)	None of these.
14	Let (X, d) be a metric space and $x \in X, 0 < r < s$. Then			
	(a)	$B(x, r) \subseteq B(x, s)$ and equality may occur.	(b)	$B(x, r) \subset B(x, s)$
	(c)	$B(x, r) = B(x, s)$ if $r \geq 1$.	(d)	None of these.
15	Consider the metric space (N, d) and (N, d_1) where d is usual distance in R and d_1 is the discrete metric in N . Then			
	(a)	The two metric spaces do not have same open balls.	(b)	The open balls in two metric spaces are same.
	(c)	Every open ball in (N, d) is an open ball in (N, d_1) .	(d)	None of these.
16	According to Hausdorff property for any two distinct points $x, y \in X$ there exists $r > 0$ such that...			
	(a)	$B(x, r) \cup B(y, r) = \emptyset$	(b)	$B(x, r) \cap B(y, r) \neq \emptyset$
	(c)	$B(x, r) \cup B(y, r) \neq \emptyset$	(d)	$B(x, r) \cap B(y, r) = \emptyset$
17	Let A be any finite set in a metric space (X, d) . Then A^c is...			
	(a)	An open set.	(b)	A closed set.
	(c)	Open as well as closed.	(d)	None of these.
18	Let (X, d) be a metric space and d_1 be the metric on X defined by $d_1(x, y) = \frac{d(x, y)}{1 + d(x, y)}$ then....			
	(a)	d and d_1 are equivalent metrics on X .	(b)	d and d_1 are not equivalent metrics on X .
	(c)	Every open ball in (X, d) is an open ball in (X, d_1) .	(d)	None of these.
19	Every closed ball in a metric space (X, d) is ...			
	(a)	A closed set.	(b)	An open set.
	(c)	Both open and closed.	(d)	None of these.
20	Very open ball in a metric space (X, d) is ...			

	(a)	A closed set.	(b)	An open set.
	(c)	Both open and closed.	(d)	None of these.

21	Let (X, d) be a metric space and $K \subseteq X$. Then			
	(a)	K is compact.	(b)	K is compact if K is closed.
	(c)	K is compact if K is bounded.	(d)	K is compact if K is finite.
22	Which of the following subsets of R^3 is compact?			
	(a)	$\{(x, y, z) \in R^3 : x^2 + y^2 - z^2 = 1\}$	(b)	$\{(x, y, z) \in R^3 : x^2 - y^2 - z^2 = 1\}$
	(c)	$\{(x, y, z) \in R^3 : x^2 + y^2 + z^2 = 1\}$	(d)	None of these.
23	In a metric space (R, d) , d is usual distance,			
	(a)	$[0, 1] \cup [2, 3]$ is compact.	(b)	$[0, 1] \cup (2, 3)$ is compact.
	(c)	$[0, 1] \cup \{x \in N : x \geq 3\}$ is compact.	(d)	$[0, 1] \cup [2, \infty)$ is compact.
24	Let A and B be compact subset of (R, d) , d is usual distance. Then the following set is not compact.			
	(a)	$A \times B$ in (R^2, d) , d being Euclidean	(b)	$A \cup B$ in R
	(c)	$A \cap B$ in R , (provided $A \cap B \neq \emptyset$)	(d)	$A \setminus B$ in R (provided $A \setminus B \neq \emptyset$)
25	Which of the following statements is false?			
	(a)	A compact subset of a metric space is closed and bounded.	(b)	A closed and bounded subset of a metric space is compact.
	(c)	A finite subset of a metric space is compact.	(d)	A closed subset of a compact set in a metric space is compact.
26	Let (X, d) be a metric space and (x_n) be a sequence in X such that $x_n \rightarrow x_0$ as $n \rightarrow \infty$. Then			
	(a)	$\{x_n : n \in N\}$ is a compact subset of X .	(b)	$\{x_n : n \in N\} \cup \{x_0\}$ is a compact subset of X .
	(c)	$\{x_n : n \in N\} \cup \{x_0\}$ is a compact subset of X only if (x_n) is a sequence of distinct points.	(d)	None of these.
27	Let A be a compact subset of R . Then			
	(a)	\bar{A} may not be compact.	(b)	A^0 may not be compact.
	(c)	∂A may not be compact.	(d)	None of these.
28	The set $\{n + \frac{1}{n} : n \in N\}$ in (R, d) , d is usual distance, is			
	(a)	Compact	(b)	Not compact.
	(c)	Connected.	(d)	None of these.
29	The set $\{(x, y) \in R^2 : x + y \leq 1\}$ as a subset of (R^2, d) , d being Euclidean distance, is			

	(a)	Compact	(b)	Not compact.
	(c)	Not Connected.	(d)	None of these.
30	If A and B are disjoint non-empty subsets of a metric space (X, d) such that A is closed and B is compact then..			
	(a)	$d(A, B) = 0$	(b)	$d(A, B) < 0$
	(c)	$d(A, B) = 1$	(d)	$d(A, B) > 0$

31	Let (X, d) be a metric space. A Sequence $\{x_n\}$ in X is said to converge to $x \in X$ if for every $\epsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that			
	(a)	$d(x_n, x) < \epsilon$ for all $n \geq n_0$	(b)	$d(x_n, x) \leq \epsilon$ for all $n \geq n_0$
	(c)	$d(x_n, x) = \epsilon$ for all $n \geq n_0$	(d)	$d(x_n, x) \geq \epsilon$ for all $n \geq n_0$
32	Let (X, d) be a metric space. A Sequence $\{x_n\}$ in X is converge to $x \in X$ if and only if the sequence $(d(x_n, x))$			
	(a)	converges to 0 in \mathbb{R}	(b)	converges to 0 in X
	(c)	diverges to 0 in \mathbb{R}	(d)	converges and diverges to 0 in \mathbb{R}
33	If X is normed linear space then (x_n) is bounded if there exists $M > 0$ such that			
	(a)	$\ x_n\ < M$ for all $n \in \mathbb{N}$	(b)	$\ x_n\ \leq M$ for all $n \in \mathbb{N}$
	(c)	$\ x_n\ = M$ for all $n \in \mathbb{N}$	(d)	$\ x_n\ \geq M$ for all $n \in \mathbb{N}$
34	Every convergent sequence in a metric space is			
	(a)	bounded	(b)	closed
	(c)	Cauchy	(d)	closed and bounded
35	Every Cauchy Sequence in a metric space is			
	(a)	bounded	(b)	closed
	(c)	convergent	(d)	convergent and bounded
36	Let (X, d) be a metric space and A be a subset of X . $p \in$ closure of A if and only if there exists a sequence of points of A			
	(a)	converging to p	(b)	converging to 0
	(c)	converging to p and 0 both	(d)	does not converge to p
37	A non empty set A is said to be countable if there exists			
	(a)	a injective function $f: A \rightarrow \mathbb{N}$	(b)	a surjective function $f: A \rightarrow \mathbb{N}$
	(c)	a bijective function $f: A \rightarrow \mathbb{N}$	(d)	a injective and surjective both function $f: A \rightarrow \mathbb{N}$
38	A metric space (X, d) is said to be separable if X has a			
	(a)	countable dense subset	(b)	convergent sequence
	(c)	uncountable dense subset	(d)	dense subset of X
39	If A and B are dense subsets of a metric space (X, d) and one of A, B is open then			

	(a)	$A \cap B$ is dense in X	(b)	$A \cup B$ is dense in X
	(c)	$A + B$ is dense in X	(d)	$A * B$ is dense in X
40	A metric space (X,d) is said to be complete if every			
	(a)	Cauchy sequence in X converges to a point in X .	(b)	a bounded sequence in X converges to a point in X .
	(c)	Closed and bounded sequence in X converges to a point in X .	(d)	subsequence in X converges to a point in X .
41	Every finite metric space is			
	(a)	complete	(b)	bounded
	(c)	closed	(d)	complete and closed
42	A Complete subspace of a metric space is			
	(a)	closed	(b)	bounded
	(c)	closed and bounded	(d)	compact
43	Bolzano Weierstrass Theorem states that			
	(a)	every bounded real sequence has a convergent subsequence	(b)	every bounded real sequence has not a convergent subsequence
	(c)	every bounded real sequence has a Cauchy sequence	(d)	every bounded real sequence and Cauchy sequence has a convergent subsequence
44	Let $f:[a,b] \rightarrow \mathbb{R}$ be a continuous real valued function. Suppose $f(a)$ and $f(b)$ are of opposite sign, then there exists $p \in [a,b]$ such that $f(p) = 0$. This statement is			
	(a)	Intermediate Value Theorem	(b)	Cauchy theorem
	(c)	Cantor's theorem	(d)	Lagrange's theorem
45	Density theorem states that			
	(a)	Let x and y be any two distinct real numbers with $x < y$, then there exists a rational number r such that $x < r < y$	(b)	Let x and y be any two distinct real numbers with $x < y$, then there exists a rational number r such that $x \leq r < y$
	(c)	Let x and y be any two distinct real numbers with $x < y$, then there exists a rational number r such that $x < r \leq y$	(d)	Let x and y be any two distinct real numbers with $x < y$, then there exists a rational number r such that $r < x < y$
46	Cantor Intersection theorem states that			
	(a)	Let (X,d) be a complete metric space. Let $\{F_n\}$, $n \in \mathbb{N}$ be a sequence of non empty closed subsets of X such that	(b)	Let (X,d) be a complete metric space. Let $\{F_n\}$, $n \in \mathbb{N}$ be a sequence of non empty closed subsets of X such that $i > F_{n+1} \subseteq F_n$ for all $n \in \mathbb{N}$

		$F_{n+1} \subseteq F_n$ for all $n \in \mathbb{N}$ ii> $\cap F_n, (n \in \mathbb{N})$ consists of exactly one point.		ii> $\cap F_n, (n \in \mathbb{N})$ consists of at least one point.
	(c)	Let (X,d) be a complete metric space. Let $\{F_n\}, n \in \mathbb{N}$ be a sequence of non empty closed subsets of X such that $F_{n+1} \subseteq F_n$ for all $n \in \mathbb{N}$ ii> $\cap F_n, (n \in \mathbb{N})$ consists of minimum one point.	(d)	Let (X,d) be a complete metric space. Let $\{F_n\}, n \in \mathbb{N}$ be a sequence of non empty closed subsets of X such that $F_{n+1} \subseteq F_n$ for all $n \in \mathbb{N}$ ii> $\cap F_n, (n \in \mathbb{N})$ consists more than point.