# Quotient Space and Orthogonal Transformations,Isometries, Cayley-Hamilton Theorem and its application

Q 1) Let $V = \mathbb{R}^3, W_1 = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 + x_2 + x_3 = 0\}$ and $W_2 = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 - x_2 + x_3 = 0\}$ are subspaces of $V$. then

(a) $dimV/W_1 = dimV/W_2 = 2, dimW_2/W_1 \cap W_2 = 1$

(b) $dimV/W_1 = dimV/W_2 = 1, dimW_2/W_1 \cap W_2 = 1$

(c) $dimV/W_1 = dimV/W_2 = 1, dimW_2/W_1 \cap W_2 = 2$

(d) None of the above.

Q 2) Let $V = M_2(\mathbb{R})$ , $W_1 =$ Space of $2\times2$ real symmetric matrices, $W_2 =$ Space of $2\times2$ real skew symmetric matrices.

(a) $dimV/W_1 = 1, dimV/W_2 = 1$ (b) $dimV/W_1 = 2, dimV/W_2 = 2$
(c) $dimV/W_1 = 1, dimV/W_2 = 3$ (d) None of the above.

Q 3) Let $V = P_2[x]$ , the space of polynomial of degree $\leq 2$ over $\mathbb{R}$ along with zero polynomial and $W = \{f \in V : f(0) = 0\}$. Then

(a) $\{\overline{1}, \overline{x+1}, \overline{(x+1)^2}\}$ is the basis of the quotient space $V/W$.

(b) $\{\overline{x+1}, \overline{x^2+1}\}$ is the basis of the quotient space $V/W$

(c) $\{\overline{x+1}\}$ is the basis of the quotient space $V/W$

(d) None of the above.

Q 4) Let $V$ be a real vector space and $T : \mathbb{R}^6 \to V$ be a linear transformation such that $S = \{Te_2, Te_4, Te_6\}$ spans $V$. Then, which of the following is true ?

(a) $S$ is a basis of $V$

(b) $\{e_1 + KerT, e_3 + KerT, e_5 + KerT\}$ is a basis of $\mathbb{R}^6/KerT$

(c) $dimV/ImT \geq 3$

(d) $dim\mathbb{R}^6/KerT \leq 3$

Q 5) Consider $W = \{(x, y, z) \in \mathbb{R}^3 : 2x+2y+z = 0, 3x+3y-2z = 0, x+y-3z = 0\}$.Then $dim\mathbb{R}^3/W$ is

(a) 1 (b) 2 (c) 3 (d) 0

Q 6) Consider the linear transformation $T : P_2[\mathbb{R}] \to M_2(\mathbb{R})$ defined by $T(f) = \begin{pmatrix} f(0) - f(2) & 0 \\ 0 & f(1) \end{pmatrix}$ where $P_2[\mathbb{R}]=$ space of polynomials of degree $\leq 2$ along with 0 polynomial.Then

(a) $kerT = 0$ and $dim(M_2(\mathbb{R})/ImT) = 3$

(b) $dim(P_2[\mathbb{R}]/KerT) = 1$

(c) $T$ is one-one and onto.

(d) $dim(P_2[\mathbb{R}]/KerT) = 2$

Q 7) Let $V = M_2(\mathbb{R})$ and $W = \left\{ A \in M_2(\mathbb{R}) : A \begin{pmatrix} 0 & 2 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 3 & 1 \end{pmatrix} A \right\}$.Then

(a) $dim V/W = 0$ (b) $dim V/W = 1$
(c) $dim V/W = 2$ (d) $dim V/W = 3$

Q 8) Let $V = \mathbb{R}^4$ and $W = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : x_1 = x_2 \text{ and } x_3 = x_4\}$ a subspace of V. Then

(a) $\{\overline{(1,1,0,0)}, \overline{(0,1,0,1)}\}$ is the basis of $V/W$.
(b) $\{\overline{(1,0,1,0)}, \overline{(0,-1,0,-1)}\}$ is the basis of $V/W$
(c) $\{\overline{(1,0,1,0)}, \overline{(0,1,0,1)}\}$ is the basis of $V/W$
(d) None of the above.

Q 9) Let $V = M_2(\mathbb{R})$.Consider the subspaces $W_1 = \left\{ \begin{pmatrix} a & -a \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{R} \right\}$ and $W_2 = \left\{ \begin{pmatrix} a & b \\ -a & d \end{pmatrix} : a, b, d \in \mathbb{R} \right\}$.Then

(a) $dim V/W_1 = dim V/W_2 = 2, dim W_2/W_1 \cap W_2 = 1$
(b) $dim V/W_1 = dim V/W_2 = 1, dim W_2/W_1 \cap W_2 = 1$
(c) $dim V/W_1 = dim V/W_2 = 1, dim W_2/W_1 \cap W_2 = 2$
(d) None of the above.

Q 10) Let $V = M_2(\mathbb{R})$ and $W = \{A \in M_2(\mathbb{R}) : Tr(A) = 0\}$ a subspace of V. Then

(a) $\left\{ \overline{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}}, \overline{\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}} \right\}$ is the basis of $V/W$. (b) $\left\{ \overline{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}} \right\}$ is the basis of $V/W$
(c) $\left\{ \overline{\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}} \right\}$ is the basis of $V/W$ (d) None of the above.

Q 11) Let $V = P_n[x]$,the space of polynomials of degree$\leq$ n over $\mathbb{R}$ along with zero polynomial and D denote the linear transformation $D : V \to P_{n-1}[x]$ defined by $D(f) = \frac{df}{dx}$. If $W = ker D$, then

(a) $dim V/W = n - 1$. (b) $dim V/W = 1$
(c) $dim V/W = n$ (d) None of these.

Q 12) Let $A$ be a $5 \times 7$ matrix over $\mathbb{R}$. Suppose rank $A = 3$.
A linear transformation $T : \mathbb{R}^7 \to \mathbb{R}^5$ is defined by$T(X) = AX$, where $X$ is a column vector in $\mathbb{R}^7$, and $W = ker T$ , $U = Img T$, then

(a) $dim \mathbb{R}^7/W = 3, dim \mathbb{R}^5/U = 2$. (b) $dim \mathbb{R}^7/W = 2, dim \mathbb{R}^5/U = 2$.
(c) $dim \mathbb{R}^7/W = 2, dim \mathbb{R}^5/U = 1$. (d) None of the above.

Q 13) Let $V = M_2(\mathbb{R})$ and $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$. A linear transformation $T : V \to V$ is defined by $T(B) = AB - B$. Then

(a) T is a linear isomorphism. (b) $dim V/ker T = 1$.
(c) $dim V/ker T = 2$. (d) None of these.

Q 14) Let $U, W$ be vector spaces over $\mathbb{R}$ with bases $\{u_1, u_2, ...., u_m\}$ and $\{w_1, w_2, ...., w_n\}$ respectively. Let $V = U \oplus V$ and linear transformation $P_U : V \to U$ be defined by $P_U(u + v) = u$, where $u \in U$ and $w \in W$. Then

(a) $dimV/kerP_U = n.$ (b) $dimV/kerP_U = m.$
(c) $dimV/kerP_U = m - n.$ (d) None of these.

Q 15) Let $V = \mathbb{R}^2, W = \{(x, y) \in \mathbb{R}^2 : y = x\}$. Then

(a) $\{\overline{(1,1)}\}$ is a bases of $V/W$. (b) $\{\overline{(1,0)}\}$ is a bases of $V/W$.
(c) $\{\overline{(1,1)}, \overline{(1,-1)}\}$ is a bases of $V/W$. (d) None of the above.

Q 16) If $\alpha : \mathbb{R}^4 \to \mathbb{R}^4$ and $\beta : \mathbb{R}^4 \to \mathbb{R}^4$ are translations such that $\alpha((1, 1, 1, 1)) = (1, 0, -1, 3)$ and $\beta((2, 2, 2, 2)) = (2, 0, 3, 4)$ then $\alpha\beta(0, 0, 0, 0)$ is
(a) $(0, 0, 0, 0)$. (b) $(0, -3, -1, 4)$. (c) $(0, 3, 1, -4)$. (d) None of these.

Q 17) If $\alpha : \mathbb{R}^2 \to \mathbb{R}^2$ be an isometry defined by $\alpha((x, y)) = (\frac{x}{2} + \frac{\sqrt{3}y}{2} - \frac{1}{2}, \frac{-\sqrt{3}x}{2} + \frac{y}{2} + \frac{\sqrt{3}}{2})$ and $\alpha((x, y)) = (\frac{\sqrt{3}}{2}, \frac{1}{2})$ then
(a) $x = 1, y = -1$. (b) $x = \sqrt{3}, y = 1$. (c) $x = 1, y = 1$. (d) None of these.

Q 18) Let $\alpha$ be an orthogonal transformation of the plane such that the matrix of $\alpha$ w. r. t. the standard basis of $\mathbb{R}^2$ is $\begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$, then $\alpha$ represents

(a) a rotation about origin through $\frac{\pi}{4}$. (b) a rotation about origin through $\frac{5\pi}{4}$.
(c) a rotation about the line $y = -x$. (d) None of the above.

Q 19) Let $\alpha : \mathbb{R}^2 \to \mathbb{R}^2$ represents the rotation about origin by angle $\frac{\pi}{4}$ and $\beta : \mathbb{R}^2 \to \mathbb{R}^2$ represents a reflection about y-axis. Then $\beta \circ \alpha$ represents

(a) a rotation about origin through angle $\frac{3\pi}{8}$. (b) reflection in the line $y = x$.
(c) a rotation about origin through angle $\frac{\pi}{8}$. (d) None of the above.

Q 20) Let $\alpha : \mathbb{R}^3 \to \mathbb{R}^3$ be an orthogonal transformation and $E = \{v \in \mathbb{R}^3 : \alpha v = v\}$. Then

(a) $dimE = 1$ (b) $dimE \geq 1$
(c) If $dimE = 2$, then $\alpha$ is reflection with respect to the plane.
(d) None of the above.

Q 21) Let $\alpha : \mathbb{R}^3 \to \mathbb{R}^3$ represents reflection in the plane $x + y + z = 0$. The matrix of $\alpha$ with respect to the standard basis of $\mathbb{R}^3$ is
(a) $\begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -1 \end{pmatrix}$ (b) $\frac{1}{3}\begin{pmatrix} 1 & -2 & -2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ (d) None of these.

Q 22) Let $V$ be an $n$-dimensional real inner product space. Suppose $B = \{e_i\}_{i=1}^n$ and $B' = \{f_i\}_{i=1}^n$ are orthogonal basis of $V$. Then

(a) If $T : V \to V$ is a linear transformation such that $T(e_i) = f_i$ for $i = 1$ to $n$, then $T$ is orthogonal.

(b) If $T : V \to V$ is a linear transformation such that $T(e_i) = f_i$ for $i = 1$ to $n$, then $T$ need not be orthogonal.

(c) There exist a linear transformation $T : V \to V$ such that $\{T(e_i)\}_{i=1}^n$ is an orthogonal basis of $V$, but $\{T(f_i)\}_{i=1}^n$ is not an orthonormal basis of $V$.

(d) None of the above.

Q 23) Let $A$ and $B$ be $n \times n$ real orthogonal matrices. Then

(a) $AB$ and $A + B$ are orthogonal matrices. (b) $AB$ and $BA$ are orthogonal matrices.
(c) $A + B$ is an orthogonal matrix. (d) None of the above.

Q 24) Let $A, B$ be $n \times n$ real matrices. If $A$ and $AB$ are orthogonal matrices,then

(a) $B$ is orthogonal but $BA$ may not be orthogonal (b) $B$ and $BA$ both are orthogonal matrices.
(c) $B$ may not be orthogonal matrix. (d) None of the above.

Q 25) Let $\alpha : \mathbb{R}^2 \to \mathbb{R}^2$ be an isometry fixing origin and $\alpha \neq$ identity. Then

(a) $\alpha((1, 0))$ is in the first quadrant. (b) $\alpha((1, 0)) \in \{(-1, 0), (0, 1), (0, -1)\}$.
(c) $\alpha((1, 0))$ lies on the unit circle $S^1$. (d) None of the above.

Q 26) If $\alpha : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation such that $\langle v, w\rangle = 0 \Rightarrow \langle \alpha(v), \alpha(w)\rangle = 0$ $\forall\ v, w \in \mathbb{R}^2$. Then

(a) $\alpha$ is an isometry of $\mathbb{R}^2$. (b) $\alpha$ is an orthogonal transformation.
(c) $\alpha = aT$ where $T$ is an orthogonal transformation and $a \in \mathbb{R}$. (d) None of the above.

Q 27) Let $\alpha : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $\alpha((x, y)) = (ax + by + e, cx + dy + f)$ where $a, b, c, d, e, f \in \mathbb{R}$. Then $\alpha$ is an isometry if and only if

(a) $ad - bc \neq 0, e, f > 0$ (b) $ad - bc = \pm 1$.
(c) $a^2 + c^2 = 1$,$b^2 + d^2 = 1, ab + cd = 0$. (d) None of the above.

Q 28) Let $V$ be a finite dimensional inner product space and $\alpha : V \to V$ be an isometry. Then

(a) $\alpha$ is one-one may not be onto. (b) $\alpha$ is one-one only if $\alpha(0) = 0$.
(c) $\alpha$ is bijective. (d) None of the above.

Q 29) Let $A = \begin{pmatrix} 10 & -9 \\ 4 & -2 \end{pmatrix}$, then

(a) $A^{-1} = \frac{1}{16}[A + 8I]$ (b) $A^{-1} = \frac{1}{16}[A - 8I]$
(c) $A^{-1} = \frac{1}{16}[-A + 8I]$ (d) $A^{-1} = \frac{1}{16}[-A - 8I]$

Q 30) The following pairs of n x n matrices do not have same characteristic polynomial.

(a) $A$ and $A^t$. (b) $A$ and $PAP^{-1}$ where $P$ is non singular $n \times n$ matrix.
(c) $A$ and $A^2$. (d) $AB$ and $BA$.

Q 31) Let $p(t) = t^2 + bt + c$ where $b, c \in \mathbb{R}$.Then the number of real matrices having $p(t)$ as characteristic polynomial is

(a) One (b) Two
(c) Infinity (d) None of the above

Q 32) Let $p(t) = t^3 - 2t^2 + 5$ be the characteristic polynomial of $A$ then det$A$ and tr$A$ are

(a) $5, -2$ (b) $2, 5$
(c) $-5, 2$ (d) $-2, 5$

Q 33) If $A$ is a $3 \times 2$ matrix over $\mathbb{R}$ and $B$ is a $2 \times 3$ matrix over $\mathbb{R}$ and $p(t)$ is the characteristic polynomial of $AB$, then

(a) $t^3$ divides $p(t)$ (b) $t^2$ divides $p(t)$
(c) $t$ divides $p(t)$ (d) None of the above

Q 34) Let $A$ and $B$ be $n \times n$ matrix over $\mathbb{R}$ such that tr$A$ = tr$B$ and det$A$ =det$B$.Then

(a) Characteristic polynomial of $A$ = Characteristic polynomial of $B$.

(b) Characteristic polynomial of $A \neq$ Characteristic polynomial of $B$.

(c) Characteristic polynomial of $A$ =Characteristic polynomial of $B$ if $n = 3$.

(d) Characteristic polynomial of $A$ =Characteristic polynomial of $B$ if $n = 2$.

Q 35) Let $A$ and $B$ be $n \times n$ matrix over $\mathbb{R}$ such that characteristic polynomial of $A$ = characteristic polynomial of $B$.Then

(a) $A$ and $B$ are similar matrices (b) det$A$ = det$B$
(c) $AB = BA$ (d) None of the above.

Q 36) Let $p(t) = t^3 - 2t^2 + 15$ be the characteristic polynomial of $A$ .Then det $A$
(a) 15 (b) -15 (c) 0 (d) None of these

Q 37) Let $A = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ (a) $A^{10} = \begin{pmatrix} 2^{10} & -2^{10} \\ -2^{10} & 2^{10} \end{pmatrix}$ (b) $A^{10} = \begin{pmatrix} 2^{11} & -2^{11} \\ -2^{11} & 2^{11} \end{pmatrix}$
(c) $A^{10} = \begin{pmatrix} 2^9 & -2^9 \\ -2^9 & 2^9 \end{pmatrix}$ (d) $A^{10} = \begin{pmatrix} -2^9 & 2^9 \\ 2^9 & -2^9 \end{pmatrix}$

Q 38) Let $A$ be a $3 \times 3$ matrix and $\lambda_1, \lambda_2$ be only two distinct eigen values of $A$.Then its characteristics polynomial $k_A(x)$ is

(a) $(x - \lambda_1)(x - \lambda_2)$

(b) $(x-\lambda_1)(x-\lambda_2)^2$

(c) $(x-\lambda_1)^2(x-\lambda_2)$

(d) $(x-\lambda_1)^2(x-\lambda_2)$ or $(x-\lambda_1)(x-\lambda_2)^2$

Q 39) Let characteristic polynomial of $A$ is $t^2+a_1t+a_0$ and and characteristic polynomial of $A^{-1}$ is $t^2+a_1^{'}t+a'_0$.Then

(a) $a_0a_0^{'}=1$ and $a_1+a_1^{'}=1$ (b) $a_1a_1^{'}=1$ and $a_0a_0^{'}=1$
(c) $a_0a_0^{'}=1$ (d) $a_0a_0^{'}=1$ and $a_1^{'}=a_1a_0^{'}$

Q 40) If $p_1(t)=t^2+a_1t+a_0$ is characteristic polynomial of $A$ and $p_2(t)=t^2+a_1^{'}t+a_0^{'}$ is characteristic polynomial of $A^2$ then

(a) $a_1^{'}=a_1^2$ and $a_0^{'}=a_0^2$ (b) $a_1^{'}=2a_1$ and $a_0^{'}=a_0^2$
(c) $a_0'=a_0^2,\ a_1'=a_1^2-2a_0$ (d) None of the above

Q 41) Let $A_{6\times 6}$ be a matrix with characteristic polynomial $x^2(x-1)(x+1)^3$, then trace $A$ and determinant of $A$ are

(a) -2, 0 (b) 2, 0 (c) 3, 1 (d) 3, 0

Q 42) $\begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix}$ and $\begin{pmatrix} a & b \\ 0 & d \end{pmatrix}$ are similar (non- zero $a, b, d$)

(a) for any reals $a, b, d$. (b) if $a=d$.
(c) if $a \neq d$. (d) never similar.

Q 43) Let $A_{6\times 6}$ be a diagonal matrix over $\mathbb{R}$ with characteristic polynomial $(x-2)^4(x+3)^2$. Let $V=\{B\in M_6(\mathbb{R}) : AB=BA\}$. Then dim $V=$

(a) 8 (b) 12 (c) 6 (d) 20.

Q 44) If $A-I_n$ is a $n\times n$ nilpotent matrix over $\mathbb{R}$, then characteristic polynomial of $A$ is

(a) $(t-1)^n$ (b) $t^n$
(c) $t^n-1$ (d) $(t^{n-1}-1)t$

Q 45) If $A\in M_2(\mathbb{R})$, $tr\,A=-1$, $det\,A=-6$ then $det\,(I_2+A)$ is

(a) -6 (b) -5 (c) -1 (d) None of the above.

Q 46) Let $A=[a_{ij}]_{10\times 10}$ be a real matrix such that $a_{i,i+1}=1$ for $1\le i\le 9$ and $a_{ij}=0$ otherwise, then

(a) $A^9(A-I)$ (b) $(A-I)^{10}$ $A^{10}=0$ $A(A-I)^9=0$

Q 47) $T:\mathbb{R}^4\to\mathbb{R}^4$ is a linear transformation such that $T^3+3T^2=4I$. If $S=T^4+3T^3-4I$, then

(a) $S$ is not one-one. (b) $S$ is one-one.

(c) if 1 is not an eigen value of $T$ then $S$ is invertible.

(d) None of these.

Q 48) Which of the following statements are true

1. If the characteristic roots of two $n \times n$ matrices are same then their characteristic polynomials are same.
2. If the characteristic polynomials of two $n \times n$ matrices are same then their characteristic roots are same.
3. If eigen values of two $n \times n$ matrices are same then their eigen vectors are same.
4. The characteristic roots of two $n \times n$ matrices are same but their characteristic polynomials may not be same.

(a) ii and iv are true. (b) i, iii are true.
(c) i, ii and iii are true. (d) only ii is true.

Q 49) A $2 \times 2$ matrix $A$ has the characteristic polynomial $x^2 + 2x - 1$, then the value of det $(2I_2 + A)$ is
(a) $\dfrac{1}{det\,A}$ (b) 0
(c) $2 + det\ A$ (d) $2\,det\ A$

Q 50) If $A$ and $B$ are $n \times n$ then trace of $I - AB + BA$ is
(a) 0 (b) $n$ (c) 2 tr $AB$ (d) None of these.

## Eigen values and Eigen vectors, Similar matrices and Minimal polynomial

Q 51) The product of all characteristic roots of a square matrix $A$ is equal to
(a) 0 (b) 1 (c) $|A|$ (d) None of these.

Q 52) If eigen value of $A$ is $\lambda$,then eigen value of $A^2$ is
(a) 1 (b) $\frac{1}{\lambda}$ (c) $\lambda^2$ (d) None of these.

Q 53) If $A$ is invertible matrix and eigen value of $A$ is $\lambda$, then eigen value of $A^{-1}$ is
(a) 1 (b) $\frac{1}{\lambda}$ (c) $\lambda$ (d) None of these.

Q 54) If the determinant of a matrix $A$ is non-zero, then its eigen values of $A$ are
(a) 1 (b) 0 (c) Non-zero (d) None of these.

Q 55) If the determinant of a matrix $A$ is zero, then one of its eigen values of $A$ is
(a) 1 (b) 0 (c) -1 (d) None of these.

Q 56) The eigen space corresponding to eigen value 1 of $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ has basis
(a) $\{(1,0)\}$ (b) $\{(1,0),(0,1)\}$
(c) $\{(0,1)\}$ (d) $\{(1,1)\}$

Q 57) Let $A = \begin{bmatrix} a & b & 1 \\ c & d & 1 \\ 1 & -1 & 0 \end{bmatrix}$ where $a, b, c, d \in \mathbb{R}$ such that $a + b = c + d$, then $A$ has eigen value
(a) $a + c$ (b) $a + b$ (c) $a - d$ (d) $b - d$

Q 58) Zero is a eigen value of a linear map $T$ from $V$ to $V$ if and only if
(a) $\text{Ker}T = \{0\}$ (b) $T$ is bijective
(c) $T$ is singular (d) $T$ is non singular

Q 59) The eigen values of a $3 \times 3$ real matrix $A$ are 1,2,3.Then

(a) Inverse of $A$ exists and it is $\frac{1}{6}(5I + 2A - A^2)$

(b) Inverse of $A$ exists and it is $\frac{1}{6}(5I + 2A + A^2)$

(c) Inverse of $A$ does not exist

(d) None of the above

Q 60) The matrix $A = \begin{pmatrix} 1 & -1 & 2 \\ 2 & -2 & 4 \\ 3 & -3 & 6 \end{pmatrix}$ has

(a) Only one distinct eigen value

(b) Only two distinct eigen values

(c) Three distinct eigen values

(d) None of the above

Q 61) The eigen vectors of the matrix $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ generate

(a) a vector space with basis $\left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\}$

(b) a vector space with basis $\left\{ \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$

(c) a vector space with basis $\left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$

(d) a vector space with basis $\left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$

Q 62) The eigen vectors of the matrix $A = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ generate a vector space of dimension

(a) 1 (b) 2 (c) 3 (d) 4

Q 63) The eigen space $E(5)$ of the matrix $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$ corresponding to the eigen value $\lambda = 5$

(a) is $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ (b) is $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$

(c) has a basis $\left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$ (d) has a basis $\left\{ \begin{pmatrix} 2 \\ -1 \end{pmatrix} \right\}$

Q 64) Let $V$ a vector space over $R$ and $I\colon V \to V$ be the identity map.Then

(a) $v$ is the only eigen vector of $I$ for some $v \in V$

(b) $2v$ is the only eigen vector of $I$ for some $v \in V$

(c) $3v$ is the only eigen vector of $I$ for some $v \in V$

(d) every vector in $V$ is an eigen vector of $I$

Q 65) Let $T\colon R^2 \to R^2$ be the linear map which rotates every vector $v \in R^2$ through an angle $\frac{\pi}{4}$.Then $T$ has

(a) no eigen vectors

(b) only two eigen vectors

(c) only three eigen vectors

(d) infinitely many eigen vectors

Q 66) Let $A_{3\times 3}$ be a real matrix of rank 1, then the eigen values of $A$ are

(a) $0, 0, 1$ (b) $0, 0, tr\, A$ (c) $0, 0, det\, A$ (d) $0, 0, -det\, A$

Q 67) Let $A = [a_{ij}]$ be a $10 \times 10$ matrix with $aij = \begin{cases} 1 & \text{if } i + j = 11 \\ 0 & \text{otherwise} \end{cases}$. Then the set of eigen values of $A$ is

(a) $\{0, 1\}$ (b) $\{1, -1\}$ (c) $\{0, 1, 10\}$ (d) $\{0, 11\}$

Q 68) Let $A_{n\times n}$ be a real matrix, then

(a) $A, A^t$ have same determinant, same eigen values and same eigen vectors.

(b) $A, A^t$ have same determinant, same eigen values but eigen vectors may be different.

(c) $A, A^t$ have same eigen values but different determinants.

(d) $A, A^t$ have different eigen values.

Q 69) Let $\sum_{j=1}^{n} a_{ij} = 1$ for a real matrix $A = [a_{ij}]$ then

(a) $(1, 1, \cdots, 1)$ is an eigen vector of $A$ corresponding to the eigen value 1.

(b) $(1, 0, \cdots, 0)$ is an eigen vector of $A$ corresponding to the eigen value 1.

(c) $(1, 1, \cdots, 1)$ is an eigen vector of $A$ corresponding to the eigen value $n$.

(d) 1 is not an eigen value of $A$.

Q 70) Let the characteristic polynomial of $A_{3\times 3}$ be $x(x-1)(x+2)$, then the characteristic polynomial of $A^2$ is
(a) $x(x+1)(x-2)$ (b) $x(x-1)(x-4)$
(c) $x(x+1)(x+4)$ (d) None of these.

Q 71) If matrix $A = \begin{bmatrix} 0 & 0 & 1 \\ a & 1 & b \\ 1 & 0 & 0 \end{bmatrix}$ has linearly independent eigen vectors corresponding to eigen value 1, then
(a) $a = 0, b = 0$. (b) $a = 1, b = 1$
(c) for any $a, b$. (d) $a + b = 0$.

Q 72) Let characteristic polynomial of $A_{2\times 2}$ be a real matrix and its characteristics polynomial is $x^2 - 3x + 2$. Then the characteristic polynomial of $A^{-1}$ is
(a) $x^2 - \frac{3}{2}x + \frac{1}{2}$ (b) $x^2 - 3x + 2$

(c) $x^2 - 2x + 3 = 0$ (d) $x^2 - \frac{1}{2}x + \frac{3}{2}$

Q 73) One of the eigen vectors of the matrix $A = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}$ over R is

(a) $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ (c) $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ (d) None of these.

Q 74) If $A$ is a square matrix of order $n$ and $\lambda$ is a scalar,then the characteristic polynomial of $A$ is obtained by expanding the determinant:
(a) $|\lambda A|$ (b) $|\lambda A - I_n|$ (c) $|A - \lambda I_n|$ (d) None of these

Q 75) At least one characteristic roots of every singular matrix is equal to
(a) 1 (b) -1 (c) 0 (d) None of these.

Q 76) The characteristic roots of two matrices $A$ and $BAB^{-1}$ are
(a) The same (b) Different (c) Always zero (d) None of these.

Q 77) The scalar $\lambda$ is a characteristic root of the matrix $A$ if:
(a) $A - \lambda I$ is non-singular (b) $A - \lambda I$ is singular (c) $A$ is singular (d) None of these.

Q 78) If eigen value of $A$ is $\lambda$,then eigen value of $P^{-1}AP$ is
(a) 1 (b) $\frac{1}{\lambda}$ (c) $\lambda$ (d) None of these.

Q 79) If $\lambda$ is a characteristic root of a matrix $A$ then characteristic roots of $-A$ and $\alpha I - A$ respectively are
(a) $-\lambda$ and $\alpha - \lambda$ (b) $-\lambda$ and $\alpha$ (c) $-\lambda$ and $\lambda$ (d) None of these.

Q 80) Which of the following statements are true

1. If the characteristic roots of two $n \times n$ matrices are same then their characteristic polynomials are same.
2. If the characteristic polynomials of two $n \times n$ matrices are same then their characteristic roots are same.
3. If eigen values of two $n \times n$ matrices are same then their eigen vectors are same.
4. The characteristic roots of two $n \times n$ matrices are same but their characteristic polynomials may not be same.

(a) ii and iv are true. (b) i, iii are true.
(c) i, ii and iii are true. (d) only ii is true.

Q 81) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the orthogonal transformation of rotation through angle $\theta$, then

(a) $T$ has no eigen values for any $\theta \in (0, 2\pi)$.
(b) $T$ has only one eigen value $-1$ for $\theta = \pi$ and no eigen values if $\theta \in (0, 2\pi) - \{\pi\}$.
(c) $T$ has eigen value 1 for $\theta = \pi/4$.
(d) $T$ has only one eigen value for all $\theta \in (0, 2\pi)$.

Q 82) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the orthogonal transformation of reflection in the line $y = \tan \frac{\theta}{2} x$, then

(a) $T$ has no eigen value for any $\theta \in (0, 2\pi)$.
(b) $T$ has only one eigen value 1 for every $\theta \in (0, 2\pi)$.
(c) $T$ has two eigen values $1, -1$ for every $\theta \in (0, 2\pi)$.
(d) $T$ has an eigen value -1 for $\theta = \pi$.

Q 83) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ where $a, b, c, d \in \mathbb{Z}$ such that $a + b = c + d$, then

(a) $A$ has two integer eigen values.
(b) $A$ may not have any eigen value.
(c) $A$ has two eigen values which may not be integers.
(d) $A$ has two eigen values only if $b, c = 0$.

Q 84) Let $A$ be an $n \times n$ orthogonal matrix with $det A = -1$. Then

(a) $-1$ is the only eigenvalue of $A$. (b) $-1$ is an eigenvalue of $A$.
(c) $A$ has at least one real eigenvalue only if $n$ is odd. (d) None of the above.

Q 85) Let $A$ be an $2 \times 2$ orthogonal matrix with $det A = 1$. Then

(a) 1 is the eigenvalue of $A$. (b) $-1$ cannot be an eigenvalue of $A$.
(c) $A$ may not have real eigenvalue. (d) None of the above.

Q 86) Let $x(x-1)(x+2)$ be the characteristic polynomial of a $3 \times 3$ matrix $A$, then the characteristic polynomial of $A^2$ is
(a) $x(x-1)(x-4)$ (b) $x(x+1)(x-2)$
(c) $x(x+1)(x+4)$ (d) None of these.

Q 87) Which of the following statements are true-

(i) 0 is an eigen value of a matrix if and only if the matrix is singular.

(ii) $A_{n \times n}$ has atleast one (real) eigen value if $n$ is odd.

(iii) A matrix with all the diagonal entries equal to zero has zero eigen value.

(iv) $det\, A =$ product of characteristic roots of $A$.

(a) all the statements are true. (b) (i), (ii), (iv) are true.

(c) (i), (iii) are true. (d) (i), (ii), (iii) are true.

Q 88) If $A$ and $B$ are $3 \times 3$ matrices over $R$ having $(1, -1, 0)^t, (1, 1, 0)^t$, and $(0, 0, 1)^t$ as eigenvectors. Then

(a) $A$ and $B$ are similar matrices. (b) $AB = BA$.

(c) $A$ and $B$ have same eigenvalues. (d) None of the above.

Q 89) If $n \times n$ real matrices $A, B$ are similar and $f(x)$ is a polynomial in real coefficients then $f(A), f(B)$ have

(a) same characteristic polynomials but different minimal polynomials.

(b) same minimal polynomial but different characteristic polynomials.

(c) same characteristic polynomial and same minimal polynomial.

(d) characteristic polynomials are different as well as the minimal polynomials are different.

Q 90) For square matrices $A, B$ of same size, which of the following statements are true?

i. If $A, B$ are similar then they have same characteristic polynomial.

ii. If $A, B$ are similar then they have same eigen vectors.

iii. If $A, B$ have same characteristic polynomial then $A, B$ are similar.

iv If $A, B$ have same characteristic roots then $A, B$ are similar.

(a) i and iv (b) only i

(c) i, ii and iv (d) None.

Q 91) The matrix $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ is

(a) similar to $\begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}$ (b) similar to $\begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix}$

(c) similar to $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (d) not similar to any diagonal matrix

Q 92) The matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ is similar to the matrix

(a) $\begin{pmatrix} 10 & -12 \\ 4 & -5 \end{pmatrix}$

(b) $\begin{pmatrix} 3 & 2 \\ 5 & -4 \end{pmatrix}$

(c) $\begin{pmatrix} 6 & 4 \\ 2 & 1 \end{pmatrix}$

(d) None of the above

Q 93) Degree of the minimal polynomial of $n \times n$ real matrix is

(a) equal to $n$. (b) less than or equal to $n$.

(c) greater than $n$. (d) less than $n$.

Q 94) Minimal polynomial of $\begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$ where $A, B$ are square matrices, is

(a) L.C.M. of the minimal polynomials of $A$ and $B$.

(b) G.C.D. of the minimal polynomials of $A$ and $B$.

(c) product of the minimal polynomials of $A$ and $B$.

(d) minimal polynomial of $A-$ minimal polynomial of $B$.

Q 95) Let $A =$ diag $\{1, 2, -1\}$, $B = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, $C = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $D = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, then

(a) $B, C, D$ are similar to $A$.

(b) Only $D$ are similar to $A$.

(c) None of $B, C, D$ are similar to $A$.

(d) $A$ is similar to $D$.

Q 96) If $A$ is a square matrix with all its eigen values equal to 1, then

(a) $A^k$ is similar to $A$ for every positive integer $k$.

(b) $A^k$ is not similar to $A$ for any positive integer $k \neq 1$.

(c) $A^k$ is similar to $A$ for only $k = 2$.

(d) $A^k = I$ for some positive integer $k$.

Q 97) The minimal polynomial of the diagonal matrix $A =$ diag $\{1, -1, 1, -1\}$ is

(a) $x^2 + 1$

(b) $x^2 - 1$

(c) $(x^2 - 1)^2$

(d) None of these.

Q 98) Let $A_{n \times n}$ be a real matrix, then the characteristic polynomial of $A =$ the minimal polynomial of $A$ if

(a) and only if $A$ has $n$ distinct characteristic roots.

(b) $A$ has $n$ distinct characteristic roots.

(c) only if $A$ is a diagonal matrix.

(d) $A$ is nilpotent matrix.

Q 99) The minimal polynomial of $\begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix}$ is

(a) $x - 1$ for any $\alpha \in \mathbb{R}$.

(b) $(x - 1)^2$ for any $\alpha \in \mathbb{R}$.

(c) $x - 1$ if $\alpha = 0$ and $(x - 1)^2$ otherwise.

(d) $x - 1$ if $\alpha \neq 0$ and $(x - 1)^2$ otherwise.

Q 100) The minimal polynomial of $\begin{bmatrix} 1 & \alpha & \beta \\ 0 & 1 & \gamma \\ 0 & 0 & 2 \end{bmatrix}$ is

(a) $(x - 1)(x - 2)$ for any $\alpha, \beta, \gamma \in \mathbb{R}$.

(b) $(x - 1)^2(x - 2)$ for any $\alpha \in \mathbb{R}$.

(c) $(x-1)^2(x-2)$ if $\alpha = 0$ and $(x-1)(x-2)$ otherwise.

(d) $(x-1)^2(x-2)$ if $\alpha \neq 0$ and $(x-1)(x-2)$ otherwise.

Q 101) If $a = \begin{bmatrix} 1 & \alpha & \beta \\ 0 & 1 & \gamma \\ 0 & 0 & 1 \end{bmatrix}$ then which of the following statements is true

(i) $x-1$ is the minimal polynomial of $A$ if and only if $\alpha = \beta = \gamma = 0$.

(ii) $(x-1)^2$ is the minimal polynomial of $A$ if and only if $\alpha = \gamma = 0$ and $\beta \neq 0$.

(iii) $(x-1)^3$ is the minimal polynomial of $A$ if and only if $\beta$ and exactly one of the $\alpha, \gamma$ are 0.

(iv) $(x-1)^3$ is the minimal polynomial of $A$ if and only if exactly two of the $\alpha, \beta, \gamma$ are 0.

(a) i, ii, iii are true. (b) only i is true.

(c) i and iii are true. (d) i, ii, iv are true.

Q 102) Let $A = \begin{bmatrix} 2 & 0 & 0 \\ a & 2 & 0 \\ b & c & -1 \end{bmatrix}$. Then $(t+1)(t-2)$ is the minimal polynomial of $A$ if and only if

(a) $b = c = 0$ (b) $a = 0$

(c) $b \neq 0$ (d) $a = b = c$.

Q 103) If $N_1, N_2$ are real nilpotent matrices , then $N_1, N_2$ are similar if and only if

(a) they have same characteristic polynomials. (b) They have same minimal polynomials.

(c) Either $N_1$ or $N_2$ is zero. (d) $N_1 = \pm N_2$

## Diagonalization of a matrix and Orthogonal Diagonalization and Quadratic Form

1. Let $A = \begin{pmatrix} 1 & 2 \\ 0 & -2 \end{pmatrix}$. Then,
   (a) $A$ and $A^{100}$ are both diagonalizable. (b) $A$ is diagonalizable but $A^{100}$ is not.
   (c) Neither $A$ nor $A^{100}$ is diagonalizable. (d) None of the above.

2. Let $A = \begin{pmatrix} 1 & 2 & 4 \\ 0 & -1 & -2 \\ 0 & 0 & 3 \end{pmatrix}$ and $B = A^{100} + A^{20} + I$. Then,
   (a) $A, B$ are not diagonalizable. (b)$A$ is diagonalizable, but $B$ is not diagonalizable.
   (c) $AB$ is diagonalizable (d) None of the above.

3. If $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation such that $T(61, 23) = (189, 93)$ and $T(67, 47) = (195, 117)$. Then
   (a) $T$ is diagonalizable with distinct eigenvalues. (b) $T$ is not diagonalizable.
   (c) $T$ does not have distinct eigenvalues, but is diagonalizable. (d) None of the above.

4. Which of the following matrices is not diagonalizable?
   (a) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

5. Let $A$ be a $n \times n$ real orthogonal matrix. Then
   (a) $A$ has $n$ real eigen values and each eigen value is $\pm 1$. (b) $A$ is diagonalizable
   (c) $A$ may not have any real eigen value. (d) (b) $A^2 = I$

6. Let $A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \end{bmatrix}$, then $A$ is diagonalizable if
   (a) $a = b, c = 1$ (b) $a = 1 = b = c$ (c) $a = b = c = 0$ (d) $a, b, c > 0$

7. Let $A = \begin{bmatrix} 0 & a \\ 0 & -a \end{bmatrix}$
   (a) $A$ is diagonalizable but not orthogonally diagonalizable.
   (b) $A$ is not diagonalizable for any $a \in \mathbb{R}$.
   (c) $A$ is orthogonally diagonalizable if and only if $a = 1$ (d) None of these.

8. If $A$ is a $4 \times 4$ matrix having all diagonal entries 0, then
   (a) 0 is an eigenvalue of $A$. (b) $A^4 = 0$ (c) $A$ is not diagonalizable. (d) None of these.

9. Let $A$ be an $n \times n$ non-zero nilpotent matrix over $\mathbb{R}$. Then
   (a) $A$ is diagonalizable. (b) $A$ is diagonalizable if $n$ is odd.
   (c) $A$ is not diagonalizable. (d) None of the above.

10. Let $A = \begin{pmatrix} \alpha & -3 \\ 3 & 0 \end{pmatrix}$, $\alpha \in \mathbb{R}$ is a parameter. Then
    (a) $A$ is not diagonalizable for any $\alpha \in \mathbb{R}$. (b) $A$ is diagonalizable $\forall \alpha \mathbb{R}$.
    (c) $A$ is not diagonalizable if $-6 \leq \alpha \leq 6$. (d) $A$ is diagonalizable if $-6 < \alpha < 6$.

11. Let $A$ and $B$ be $n \times n$ matrices over $\mathbb{R}$ such that $AB = A - B$. If $B$ is a diagonalizable matrix with only one eigenvalue 2, then,
   (a) 2 is also an eigenvalue of $A$. (b) $A$ is diagonalizable and $-2$ is the only eigenvalue of $A$.
   (c) $A$ may not be diagonalizable. (d) None of these.

12. The matrix $A = \begin{pmatrix} 1 & 7 & 5 \\ 0 & 4 & 7 \\ 0 & 0 & 2 \end{pmatrix}$

   (a) Not diagonizable. (b) is similar to $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

   (c) is similar to $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$. (d) None of the above.

13. Let $A, B, C$ be $3 \times 3$ non-diagonal matrices over $\mathbb{R}$ such that $A^2 = A, B^2 = -I, (C - 3I)^2 = 0$. Then
   (a) $A, B, C$ are all diagonalizable over $\mathbb{R}$. (b) $A, C$ are all diagonalizable over $R$.
   (c) Only $A$ is diagonalizable over $\mathbb{R}$. (d) None of the above

14. Let $A \in M_3(\mathbb{R})$ such that $AB = BA$ for all $B \in M_3(\mathbb{R})$ . Then
   (a) $A$ has distinct eigenvalues and is diagonalizable.
   (b) $A$ is not diagonalizable.
   (c) $A$ does not have distinct eigenvalues but is diagonalizable.
   (d) None of the above.

15. If $A, B, C, D \in M_2(\mathbb{R})$ such that $A, B, C, D$ are non-zero and not diagonal. If $A^2 = I, B^2 = B, C^2 = 0, C \neq 0$ and every eigenvalue of $D$ is 2, then
   (a) $A, B, C, D$ are all diagonalizable. (b) $B, C, D$ are diagonalizable.
   (c) $A, B$ are diagonalizable. (d) Only $D$ is diagonalizable.

16. If $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ then

   (a) Both $A, B$ are diagonalizable, $A$ is also orthogonally diagonalizable.
   (b) Both $A, B$ are orthogonally diagonalizable.
   (c) Both $A, B$ are diagonalizable, $B$ is also orthogonally diagonalizable.
   (d) Both $A, B$ are diagonalizable, but both $A, B$ are not orthogonally diagonalizable.

17. If $v = [1, 0, 1]$ is a row vector then,
   (a) $v^t v$ is not orthogonally diagonalizable.
   (b) $vv^t v$ is orthogonally diagonalizable.

   (c) $v^t v$ is not diagonalizable.
   (d) None of the above.

18. Let $A$ be an $m \times n$ matrix over $\mathbb{R}$. Then
(a) $AA^t$ is not orthogonally diagonalizable.
(b) $I_m + AA^t$ is not orthogonally diagonalizable.
(c) $AA^t$ and $A^tA$ are orthogonally diagonalizable. (d) None of the above.

19. Let $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$. If $P^tAP = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$, then $P =$
(a) $\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$ (b) $\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ (c) $\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ (d) None of the above.

20. Let $A = \begin{pmatrix} 0 & a \\ -a & 0 \end{pmatrix}$, $a \in \mathbb{R}$. Then
(a) $A$ is not diagonalizable for any $a \in \mathbb{R}$.
(b) $A$ is diagonalizable but not orthogonally diagonalizable.
(c) $A$ is orthogonally diagonalizable if and only if $a = 0$. (d) None of the above.

21. The equation $2x^2 - 4xy - y^2 - 4x + 10y - 13 = 0$ after rotation and translation can be reduced to
(a) an ellipse (b) a hyperbola (c) a parabola (d) a pair of straight lines.

22. The conic $x^2 + 2xy + y^2 = 1$ reduces to the standard form after rotation through a angle
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{2\pi}{3}$ (d) $\frac{\pi}{6}$

23. The quadratic form $Q(x) = x_1^2 + 4x_1x_2 + x_2^2$ has
(a) rank = 1, signature = 1. (b) rank = 2, signature = 0.
(c) rank = 2, signature = 2. (d) None of the above.

24. Let $A$ be a $4 \times 4$ real symmetric matrix. Then there exists a $4 \times 4$ real symmetric matrix $B$ such that
(a) $B^2 = A$ (b) $B^3 = A$ (c) $B^4 = A$ (d) None of these

25. The matrix $\begin{pmatrix} 1 & 2 \\ 2 & k \end{pmatrix}$ is positive definite if
(a) $k > 4$ (b) $-2 < k < 2$ (c)$|k| > 2$ (d) None of these.

26. $ax^2 + bxy + cy^2 = d$ where $a, b, c$ are not all zero and $d > 0$ represents
(a) ellipse if $b^2 - 4ac > 0$ and hyperbola if $b^2 - 4ac < 0$.
(b) ellipse if $b^2 - 4ac < 0$ and hyperbola if $b^2 - 4ac > 0$.
(c) is a circle if $b = 0$ and $a = c$ else it is a hyperbola.
(d) None of these.

27. The conic $x^2 + 10x + 7y = -32$ represents
(a) a hyperbola (b) an ellipse. (c) a parabola (d) a pair of straight lines.

28. For the quadratic from $Q(x) = 2x_1^2 + 2x_2^2 - 2x_1x_2$
(a) rank = 2, signature = 1 (b) rank = 1, signature = 1
(c) rank = 2, signature = 0 (d) rank = 2, signature = 2

29. For the quadratic from $Q(x) = -3x_1^2 + 5x_2^2 + 2x_1x_2$,
   (a) rank = 2, signature = 0 (b) rank = 2, signature = 1
   (c) rank = 2, signature = 2 (d) rank = 1, signature = 1

30. The symmetric matrix associated to the quadratic from $5(x_1 - x_2)^2$ is,
   (a) positive definite (b) positive semi definite (b) indefinite (d) negative definite.

31. The quadratic form $Q(x) = 2x_1^2 - 4x_1x_2 - x_2^2$ after rotation can be reduced to standard form
   (a) $3y_1^2 - 2y_2^2$ or $2y_1^2 + 3y_2^2$ (b) $3y_1^2 + 2y_2^2$ (c) $-3y_1^2 + 2y_2^2$ (d) $2y_1^2 - 4y_2^2$

32. The equation $x^2 + y^2 + z^2 - 2x + 4y - 6z = 11$ represents
   (a) None of the below (b) a hyperboloid of one sheet
   (c) a hyperboloid of two sheet (d) a sphere.

33. The conic $3x^2 - 4xy = 2$ represents
   (a) an ellipse (b) a hyperbola (c) a parabola (d) a pair of straight lines.

34. Let $Q(X) = X^tAX$, where $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, $X = (x_1, x_2, x_3, x_4)^t$. Then by orthogonal change of variable, $Q(X)$ can be reduced to
   (a) $y_1y_2 + y_3^2$ (b) $y_1y_2 + y_2^2 + y_3^2$
   (c) $y_1^2 + y_2^2 + y_3^2 - y_4^2$ (d) $y_2^2 + y_2^2 - y_3y_4$

35. If $A_{n\times n}$ be real matrix then which of the following is true-
   (a) $A$ has at least one eigen value. (b) $\forall X, Y \in \mathbb{R}$, $\langle AX, AY \rangle > 0$
   (c) Each eigen value of $A^tA \geq 0$ (d) $A^tA$ has $n$ eigen values.